Sail Manual

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January 30, 2017

Contents

1 Sail syntax

l	::=			Source location
	|			
annot	::=			
id	::=			Identifier
	|	x		
	|	(**deinfix** x)		remove infix status
	|	**bool**	M	Built in type identifiers
	|	**bit**	M	
	|	**unit**	M	
	|	**nat**	M	
	|	**string**	M	
	|	**range**	M	
	|	**atom**	M	
	|	**vector**	M	
	|	**list**	M	
	|	**set**	M	
	|	**reg**	M	
	|	*to_num*	M	Built in function identifiers
	|	*to_vec*	M	
	|	**msb**	M	
kid	::=			variables with kind, ticked to differntiate from program variables
	|	'x		
base_kind	::=			base kind
	|	**Type**		kind of types

	|	**Nat**	kind of natural number size expressions
	|	**Order**	kind of vector order specifications
	|	**Effect**	kind of effect sets

kind	::=		kinds
	|	$base\_kind_1 \rightarrow \ ... \ \rightarrow base\_kind_n$	

nexp	::=		expression of kind Nat, for vector sizes and origins
	|	*id*	identifier, bound by def Nat x = nexp
	|	*kid*	variable
	|	*num*	constant
	|	$nexp_1 * nexp_2$	product
	|	$nexp_1 + nexp_2$	sum
	|	$nexp_1 - nexp_2$	subtraction
	|	$2 * *nexp$	exponential
	|	**neg** *nexp*	For internal use
	|	(*nexp*) S	

order	::=		vector order specifications, of kind Order
	|	*kid*	variable
	|	**inc**	increasing (little-endian)
	|	**dec**	decreasing (big-endian)
	|	(*order*) S	

base_effect	::=		effect
	|	**rreg**	read register
	|	**wreg**	write register
	|	**rmem**	read memory
	|	**wmem**	write memory
	|	**wmea**	signal effective address for writing memory

	wmv		write memory, sending only value
	barr		memory barrier
	depend		dynamic footprint
	undef		undefined-instruction exception
	unspec		unspecified values
	nondet		nondeterminism from intra-instruction parallelism
	escape		Tracking of expressions and functions that might call exit
	lset		Local mutation happend; not user-writable
	lret		Local return happened; not user-writable

effect	::=		effect set, of kind Effects
	kid		
	$\{base\_effect_1, .., base\_effect_n\}$		effect set
	pure	M	sugar for empty effect set
	$effect_1 \uplus .. \uplus effect_n$	M	meta operation for combining sets of effects

typ	::=		Type expressions, of kind **Type**
	_		Unspecified type
	id		Defined type
	kid		Type variable
	$typ_1 \rightarrow typ_2$ **effect** *effect*		Function type (first-order only in user code)
	$(typ_1, ..., typ_n)$		Tuple type
	$id\langle typ\_arg_1, .., typ\_arg_n\rangle$		type constructor application
	(typ)	S	
	$[\|nexp\|]$	S	sugar for `range<0, nexp>`
	$[\|nexp : nexp'\|]$	S	sugar for `range< nexp, nexp'>`
	$[: nexp :]$	S	sugar for `atom<nexp>` which is special case of `range<nexp,nexp>`
	$typ[nexp]$	S	sugar for vector indexed by [*nexp*]
	$typ[nexp : nexp']$	S	sugar for vector indexed by [$nexp..nexp'$]
	$typ[nexp <: nexp']$	S	sugar for increasing vector indexed as above

	$typ[nexp :> nexp']$	S	sugar for decreasing vector indexed as above

typ_arg	::=		Type constructor arguments of all kinds
	|	*nexp*	
	|	*typ*	
	|	*order*	
	|	*effect*	

n_constraint	::=		constraint over kind **Nat**
	|	$nexp = nexp'$	
	|	$nexp \geq nexp'$	
	|	$nexp \leq nexp'$	
	|	$kid\ \mathbf{IN}\ \{num_1, \ldots, num_n\}$	

kinded_id	::=		optionally kind-annotated identifier
	|	*kid*	identifier
	|	*kind kid*	kind-annotated variable

quant_item	::=		Either a kinded identifier or a nexp constraint for a typquant
	|	*kinded_id*	An optionally kinded identifier
	|	*n_constraint*	A constraint for this type

typquant	::=		type quantifiers and constraints
	|	$\mathbf{forall}\ quant\_item_1, \ldots, quant\_item_n.$	
	|		sugar, omitting quantifier and constraints

typschm	::=		type scheme
	|	*typquant typ*	

name_scm_opt	::=		Optional variable-naming-scheme specification for variables of defined type
	|		
	|	[**name** = *regexp*]	
type_def	::=		Type definition body
	|	**typedef** *id name_scm_opt* = *typschm*	type abbreviation
	|	**typedef** *id name_scm_opt* = **const struct** *typquant*{$typ_1\ id_1$; ...; $typ_n\ id_n$;?}	struct type definition
	|	**typedef** *id name_scm_opt* = **const union** *typquant*{$type\_union_1$; ...; $type\_union_n$;?}	union type definition
	|	**typedef** *id name_scm_opt* = **enumerate** {id_1; ...; id_n ;?}	enumeration type definition
	|	**typedef** *id* = **register bits** [*nexp* : *nexp′*]{$index\_range_1$: id_1; ...; $index\_range_n$: id_n}	register mutable bitfield type definition
type_union	::=		Type union constructors
	|	*id*	
	|	*typ id*	
index_range	::=		index specification, for bitfields in register types
	|	*num*	single index
	|	$num_1..num_2$	index range
	|	$index\_range_1, index\_range_2$	concatenation of index ranges
lit	::=		Literal constant
	|	()	() : **unit**
	|	**bitzero**	**bitzero** : **bit**
	|	**bitone**	**bitone** : **bit**
	|	**true**	**true** : **bool**

	false	**false** : **bool**
	num	natural number constant
	hex	bit vector constant, C-style
	bin	bit vector constant, C-style
	undefined	constant representing undefined values
	string	string constant

$;^?$	::=		Optional semi-colon
	|		
	|	;	

pat	::=		Pattern
	|	*lit*	literal constant pattern
	|	_	wildcard
	|	(*pat* **as** *id*)	named pattern
	|	(*typ*)*pat*	typed pattern
	|	*id*	identifier
	|	$id(pat_1, .., pat_n)$	union constructor pattern
	|	$\{fpat_1; ...; fpat_n ;^?\}$	struct pattern
	|	$[pat_1, .., pat_n]$	vector pattern
	|	$[num_1 = pat_1, .., num_n = pat_n]$	vector pattern (with explicit indices)
	|	$pat_1 : .... : pat_n$	concatenated vector pattern
	|	$(pat_1, ..., pat_n)$	tuple pattern
	|	$[\|\|pat_1, .., pat_n\|\|]$	list pattern
	|	(*pat*)	S

fpat	::=		Field pattern
	|	$id = pat$	

exp	::=		Expression

	$\{exp_1; \ldots; exp_n\}$	block
	nondet $\{exp_1; \ldots; exp_n\}$	nondeterminisitic block, expressions evaluate in an unspecified order, or concurrently
	id	identifier
	lit	literal constant
	$(typ)exp$	cast
	$id(exp_1, .., exp_n)$	function application
S	$id\ exp$	No extra parens needed when exp is a tuple
	$exp_1\ id\ exp_2$	infix function application
	$(exp_1, ..., exp_n)$	tuple
	if exp_1 **then** exp_2 **else** exp_3	conditional
S	**if** exp_1 **then** exp_2	
	foreach $(id$ **from** exp_1 **to** exp_2 **by** exp_3 **in** $order)exp_4$	loop
S	**foreach** $(id$ **from** exp_1 **to** exp_2 **by** $exp_3)exp_4$	
S	**foreach** $(id$ **from** exp_1 **to** $exp_2)exp_3$	
S	**foreach** $(id$ **from** exp_1 **downto** exp_2 **by** $exp_3)exp_4$	
S	**foreach** $(id$ **from** exp_1 **downto** $exp_2)exp_3$	
	$[exp_1, \ldots, exp_n]$	vector (indexed from 0)
	$[num_1 = exp_1, \ldots, num_n = exp_n\ opt\_default]$	vector (indexed consecutively)
	$exp[exp']$	vector access
	$exp[exp_1..exp_2]$	subvector extraction
	$[exp$ **with** $exp_1 = exp_2]$	vector functional update
	$[exp$ **with** $exp_1 : exp_2 = exp_3]$	vector subrange update (with vector)
	$exp : exp_2$	vector concatenation
	$[\|\|exp_1, .., exp_n\|\|]$	list
	$exp_1 :: exp_2$	cons
	$\{fexps\}$	struct
	$\{exp$ **with** $fexps\}$	functional update of struct
	$exp.id$	field projection from struct
	switch $exp\{$ **case** $pexp_1 \ldots$ **case** $pexp_n\}$	pattern matching

		letbind **in** *exp*	let expression
		lexp := *exp*	imperative assignment
		sizeof *nexp*	Expression to return the value of the nexp variable or expression at run time
		exit *exp*	expression to halt all current execution, potentially calling a system, trap, or interrupt handler with exp
		return *exp*	expression to end current function execution and return the value of exp from the function; this can be used to break out of
		assert (*exp*, *exp′*)	expression to halt with error, when the first expression is false, reporting the optional string as an error
		(*exp*)	S
		(*annot*)*exp*	This is an internal cast, generated during type checking that will resolve into a syntactic cast after
		annot	This is an internal use for passing nexp information to library functions, postponed for constraint solving
		sizeof *annot*	For sizeof during type checking, to replace nexp with internal n
		annot, *annot′*	This is like the above but the user has specified an implicit parameter for the current function
		comment *string*	For generated unstructured comments
		comment *exp*	For generated structured comments
		let *lexp* = *exp* **in** *exp′*	This is an internal node for compilation that demonstrates the scope of a local mutable variable
		let *pat* = *exp* **in** *exp′*	This is an internal node, used to distinguised some introduced lets during processing from original ones
		return_int(*exp*)	For internal use to embed into monad definition

lexp	::=		lvalue expression
		id	identifier
		id(exp_1, .. , exp_n)	memory write via function call
		id exp	S
		(*typ*)*id*	
		($lexp_0$, .. , $lexp_n$)	set multiple at a time, a check will ensure it's not memory
		lexp[*exp*]	vector element
		lexp[exp_1..exp_2]	subvector
		lexp.id	struct field

fexp	::=		Field-expression
		id = *exp*	

fexps	::=		Field-expression list
	|	$fexp_1$; ... ; $fexp_n$;$^?$	
opt_default	::=		Optional default value for indexed vectors, to define a defualt value for any unspecified positions in a sparse map
	|		
	|	; **default** = *exp*	
pexp	::=		Pattern match
	|	*pat* → *exp*	
tannot_opt	::=		Optional type annotation for functions
	|	*typquant typ*	
rec_opt	::=		Optional recursive annotation for functions
	|		non-recursive
	|	**rec**	recursive
effect_opt	::=		Optional effect annotation for functions
	|		sugar for empty effect set
	|	**effect** *effect*	
funcl	::=		Function clause
	|	*id pat* = *exp*	
fundef	::=		Function definition
	|	**function** *rec_opt tannot_opt effect_opt* $funcl_1$ **and** ... **and** $funcl_n$	
letbind	::=		Let binding
	|	**let** *typschm pat* = *exp*	value binding, explicit type (*pat* must be total)

	| **let** *pat* = *exp*	value binding, implicit type (*pat* must be total)
val_spec	::=	Value type specification
	| **val** *typschm id*	
	| **val extern** *typschm id*	
	| **val extern** *typschm id* = *string*	Specify the type and id of a function from Lem, where the string must provide an explicit path to the requ
default_spec	::=	Default kinding or typing assumption
	| **default** *base_kind kid*	
	| **default Order** *order*	
	| **default** *typschm id*	
scattered_def	::=	Function and type union definitions that can be spread across a file. Each one must end in **id**
	| **scattered function** *rec_opt tannot_opt effect_opt id*	scattered function definition header
	| **function clause** *funcl*	scattered function definition clause
	| **scattered typedef** *id name_scm_opt* = **const union** *typquant*	scattered union definition header
	| **union** *id* **member** *type_union*	scattered union definition member
	| **end** *id*	scattered definition end
reg_id	::=	
	| *id*	
alias_spec	::=	Register alias expression forms. Other than where noted, each id must refer to an unaliased register of type v
	| *reg_id*.*id*	
	| *reg_id*[*exp*]	
	| *reg_id*[*exp*..*exp*′]	
	| *reg_id* : *reg_id*′	

dec_spec	::=		Register declarations
	|	**register** *typ id*	
	|	**register alias** *id* = *alias_spec*	
	|	**register alias** *typ id* = *alias_spec*	
def	::=		Top-level definition
	|	*kind_def*	definition of named kind identifiers
	|	*type_def*	type definition
	|	*fundef*	function definition
	|	*letbind*	value definition
	|	*val_spec*	top-level type constraint
	|	*default_spec*	default kind and type assumptions
	|	*scattered_def*	scattered function and type definition
	|	*dec_spec*	register declaration
	|	*dec_comm*	generated comments
defs	::=		Definition sequence
	|	def_1 .. def_n	

2 Sail primitive types and functions

built_in_types	::=		Type Kind
	|	**bit** : **Typ**	
	|	**unit** : **Typ**	
	|	**forall Nat** $'n$. **Nat** $'m$. **range** $<'$ n,$'$ m $>$: **Nat** $\rightarrow$ **Nat** $\rightarrow$ **Typ**	
	|	**forall Nat** $'n$. **atom** $<'$ n $>$: **Nat** $\rightarrow$ **Typ**	singleton number, instead of range$<'$ n,$'$ n
	|	**forall Nat** $'n$, **Nat** $'m$, **Order** $'o$, **Typ** $'t$. **vector** $\langle' n$,$'$ m,$'$ o,$'$ $t\rangle$: **Nat** $\rightarrow$ **Nat** $\rightarrow$ **Order** $\rightarrow$ **Typ**	
	|	**forall Typ** $'a$. **option** $\langle' a\rangle$: **Typ** $\rightarrow$ **Typ**	
	|	**forall Typ** $'t$. **register** $\langle' t\rangle$: **Typ** $\rightarrow$ **Typ**	
	|	**forall Typ** $'t$. **reg** $\langle' t\rangle$: **Typ** $\rightarrow$ **Typ**	internal reference cell
	|	**forall Nat** $'n$. **implicit** $<'$ n $>$: **Nat** $\rightarrow$ **Typ**	To add to a function val specification ind

built_in_type_abbreviations	::=	
	|	**bool** $\Rightarrow$ **bit**
	|	**nat** $\Rightarrow$ $[\vert 0..pos\_infinity \vert]$
	|	**int** $\Rightarrow$ $[\vert neg\_infinity..pos\_infinity \vert]$
	|	$uint8 \Rightarrow [\vert 0..2 * *8 \vert]$
	|	$uint16 \Rightarrow [\vert 0..2 * *16 \vert]$
	|	$uint32 \Rightarrow [\vert 0..2 * *32 \vert]$
	|	$uint64 \Rightarrow [\vert 0..2 * *64 \vert]$

functions	::=		Built-in functions: all have effect pure, all order polymorphic
	|	**val forall Typ** $'a$.$'a$ $\rightarrow$ **unit** : **ignore**	
	|	**val forall Typ** $'a$.$'a$ $\rightarrow$ **option** $\langle' a\rangle$: **Some**	
	|	**val forall Typ** $'a$. **unit** $\rightarrow$ **option** $\langle' a\rangle$: **None**	
	|	**val** ([:$'$ n :], [:$'$ m :]) $\rightarrow$ $[\vert' n +' m \vert]$: $+$	arithmetic addition
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) $\rightarrow$ **bit** $['n]$: $+$	unsigned vector addition
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) $\rightarrow$ (**bit** $['n]$, **bit** , **bit**) : $+$	unsigned vector addition with overflow, carry out
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) $\rightarrow$ **bit** $['n]$: $+\_s$	signed vector addition
	|	**val forall Nat** $'n$.($bit['n]$, **bit** $['n]$) $\rightarrow$ (**bit** $['n]$, **bit** , **bit**) : $+\_s$	signed vector addition with overflow, carry out

$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to [|'n -' o..'m -' p|] : -$	arithmetic subtraction
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : -$	unsigned vector subtraction
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to (\,\mathbf{bit}\,['n],\,\mathbf{bit}\,,\,\mathbf{bit}\,) : -$	unsigned vector subtraction with overflow, carry out
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : -\_s$	signed vector subtraction
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(bit['n],\,\mathbf{bit}\,['n]) \to (\,\mathbf{bit}\,['n],\,\mathbf{bit}\,,\,\mathbf{bit}\,) : -\_s$	signed vector subtraction with overflow, carry out
$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to [|'n *' o..'m *' p|] : *$	arithmetic multiplication
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *$	unsigned vector multiplication
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *\_s$	signed vector multiplication
$\mathbf{val}\,([|'n..'m|],[|1..'p|]) \to [|0..'p-1|] : \mathbf{mod}$	arithmetic modulo
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : \mathbf{mod}$	unsigned vector modulo
$\mathbf{val}\,([|'n..'m|],[|1..'p|]) \to [|'q..'r|] : \mathbf{quot}$	arithmetic integer division
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] : \mathbf{quot}$	unsigned vector division
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] : quot\_s$	signed vector division
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\,\mathbf{Nat}\,'n.('a['n] \to [:'\,n\,:]) : \mathbf{length}$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,'\,n \leq' m.(\,\mathbf{implicit}\,\langle'm\rangle,'\,a['n]) \to' a['m] : \mathbf{mask}$	reduce size of vector, dropping MSBits. Type system supplies implicit p
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\equiv$	vector equality
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\,\mathbf{Typ}\,'b.('a,'\,b) \to \mathbf{bit}\ :\equiv$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\,\mathbf{Typ}\,'b.('a,'\,b) \to \mathbf{bit}\ :!=$	
$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to \mathbf{bit}\ :\langle$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\langle$	unsigned less than
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :<\_s$	
$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to \mathbf{bit}\ :\rangle$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\rangle$	unsigned greater than
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :>\_s$	
$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to \mathbf{bit}\ :\leq$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\leq$	unsigned less than or eq
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :<=\_s$	
$\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to \mathbf{bit}\ :\geq$	
$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\geq$	unsigned greater than or eq

	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :>=\ \_s$	
	$\mathbf{val}\,\mathbf{bit}\ \to\ \mathbf{bit}\ :$	bit negation
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.\,\mathbf{bit}\,['n] \to \mathbf{bit}\,['n] :$	bitwise negation
	$\mathbf{val}\,(\,\mathbf{bit}\,,\,\mathbf{bit}\,) \to \mathbf{bit}\ :\mid$	bitwise or
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] :\mid$	
	$\mathbf{val}\,(\,\mathbf{bit}\,,\,\mathbf{bit}\,) \to \mathbf{bit}\ :\&$	bitwise and
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] :\&$	
	$\mathbf{val}\,(\,\mathbf{bit}\,,\,\mathbf{bit}\,) \to \mathbf{bit}\ :\uparrow$	bitwise xor
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] :\uparrow$	
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,,[|'n|]) \to \mathbf{bit}\,['n] :\uparrow\uparrow$	duplicate bit into a vector
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,'\,m \leq' n.(\,\mathbf{bit}\,['n],[|'m|]) \to \mathbf{bit}\,['n] :<<$	left shift
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,'\,m \leq' n.(\,\mathbf{bit}\,['n],[|'m|]) \to \mathbf{bit}\,['n] :>>$	right shift
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,'\,m \leq' n.(\,\mathbf{bit}\,['n],[|'m|]) \to \mathbf{bit}\,['n] :<<<$	rotate

functions_with_coercions ::=

	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(bit['n],\,\mathbf{bit}\,['n]) \to [|2 **' n|] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to \mathbf{bit}\,['n] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.([|'o..'p|],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *' n|] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,) \to \mathbf{bit}\,['n] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n(\,\mathbf{bit}\,,\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : +$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(bit['n],\,\mathbf{bit}\,['n]) \to [|2 **' n|] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to \mathbf{bit}\,['n] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.([|'o..'p|],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *' n|] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,) \to \mathbf{bit}\,['n] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n(\,\mathbf{bit}\,,\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : +\_s$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to \mathbf{bit}\,['n] : -$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.([|'o..'p|],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : -$
	$\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'o,\,\mathbf{Nat}\,'p.(\,\mathbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *' n|] : -$

3 Sail type system

3.1 Internal type syntax

k	::=		Internal kinds
	|	K_Typ	
	|	K_Nat	
	|	K_Ord	
	|	K_Efct	
	|	$K\_Lam(k_0 .. k_n \to k')$	
	|	K_infer	Representing an unknown kind, inferred by context

$t,\ u$	::=			Internal types
	|	x		
	|	$'x$		
	|	$t_1 \to t_2$ *effect*		
	|	$(t_1, ..., t_n)$		
	|	$x\langle t\_args\rangle$		
	|	$t \mapsto t_1$		
	|	**register** $\langle t\_arg\rangle$	S	
	|	**range** $\langle ne\ ne'\rangle$	S	
	|	**atom** $\langle ne\rangle$	S	
	|	**vector** $\langle ne\ ne'\ order\ t\rangle$	S	
	|	**list** $\langle t\rangle$	S	
	|	**reg** $\langle t\rangle$	S	
	|	**implicit** $\langle ne\rangle$	S	
	|	**bit**	S	
	|	*string*	S	
	|	**unit**	S	
	|	$t[t\_arg_1/tid_1 .. t\_arg_n/tid_n]$	M	

optx	::=		
	|	x	
	|		
tag	::=		Data indicating where the identifier arises and thus information necessary in compilation
	|	**None**	
	|	**Intro**	Denotes an assignment and lexp that introduces a binding
	|	**Set**	Denotes an expression that mutates a local variable
	|	**Tuple**	Denotes an assignment with a tuple lexp
	|	**Global**	Globally let-bound or enumeration based value/variable
	|	**Ctor**	Data constructor from a type union
	|	**Extern** *optx*	External function, specied only with a val statement
	|	**Default**	Type has come from default declaration, identifier may not be bound locally
	|	**Spec**	
	|	**Enum** *num*	
	|	**Alias**	
	|	$Unknown\_path optx$	Tag to distinguish an unknown path from a non-analysis non deterministic path
ne	::=		internal numeric expressions
	|	x	
	|	$'x$	
	|	*num*	
	|	**infinity**	
	|	$ne_1 * ne_2$	
	|	$ne_1 + ... + ne_n$	
	|	$ne_1 - ne_2$	
	|	$2 ** ne$	
	|	$(-ne)$	
	|	**zero**	S
	|	**one**	S

	bitlength (bin)	M	
	bitlength (hex)	M	
	count $(num_0 \ldots num_i)$	M	
	length $(pat_1 \ldots pat_n)$	M	
	length $(exp_1 \ldots exp_n)$	M	

t_arg	::=		Argument to type constructors
	t		
	ne		
	effect		
	order		
	fresh	M	

t_args	::=		Arguments to type constructors
	$t\_arg_1 \ldots t\_arg_n$		

nec	::=		Numeric expression constraints
	$ne \leq ne'$		
	$ne = ne'$		
	$ne \geq ne'$		
	$'x$ **IN** $\{num_1, \ldots, num_n\}$		
	$nec_0 \,..\, nec_n \rightarrow nec'_0 \ldots nec'_m$		
	$nec_0 \ldots nec_n$		

Σ^{N}	::=		nexp constraint lists
	$\{nec_1, \,..\, , nec_n\}$		
	${\Sigma^{\mathrm{N}}}_1 \uplus \,..\, \uplus {\Sigma^{\mathrm{N}}}_n$	M	
	consistent_increase $ne_1 \; ne'_1 \ldots ne_n \; ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always larger than the su
	consistent_decrease $ne_1 \; ne'_1 \ldots ne_n \; ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always smaller than the
	resolve (Σ^{N})		

E^{D}	::=			Environments storing top level information, such as defined abbreviations, records, enumerations, and kinds
	|	$\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle$		
	|	ϵ		
	|	$E^{\mathrm{D}} \uplus E^{\mathrm{D}\prime}$		
kinf	::=			Whether a kind is default or from a local binding
	|	k		
	|	k **default**		
tid	::=			A type identifier or type variable
	|	*id*		
	|	*kid*		
E^{K}	::=			Kind environments
	|	$\{tid_1 \mapsto kinf_1, .., tid_n \mapsto kinf_n\}$		
	|	$E^{\mathrm{K}}{}_1 \uplus .. \uplus E^{\mathrm{K}}{}_n$	M	In a unioning kinf, k default u k results in k (i.e. the default is locally forgotten)
	|	$E^{\mathrm{K}} \setminus E^{\mathrm{K}}{}_1 .. E^{\mathrm{K}}{}_n$	M	
tinf	::=			Type variables, type, and constraints, bound to an identifier
	|	t		
	|	$E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t$		
E^{A}	::=			
	|	$\{tid_1 \mapsto tinf_1, .., tid_n \mapsto tinf_n\}$		
	|	$E^{\mathrm{A}}_1 \uplus .. \uplus E^{\mathrm{A}}_n$		
field_typs	::=			Record fields
	|	$id_1 : t_1, .., id_n : t_n$		
E^{R}	::=			Record environments

	|	$\{\{field\_typs_1\} \mapsto tinf_1, .., \{field\_typs_n\} \mapsto tinf_n\}$	
	|	$E^{\mathrm{R}}_1 \uplus .. \uplus E^{\mathrm{R}}_n$	M
enumerate_map	::=		
	|	$\{num_1 \mapsto id_1 \; ... \; num_n \mapsto id_n\}$	
E^{E}	::=		Enumeration environments
	|	$\{t_1 \mapsto enumerate\_map_1, .., t_n \mapsto enumerate\_map_n\}$	
	|	$E^{\mathrm{E}}_1 \uplus .. \uplus E^{\mathrm{E}}_n$	
E^{T}	::=		Type environments
	|	$\{id_1 \mapsto tinf_1, .., id_n \mapsto tinf_n\}$	
	|	$\{id \mapsto \textbf{overload}\ tinf\ conformsto : tinf_1, \; ... \; , tinf_n\}$	
	|	$(E^{\mathrm{T}}{}_1 \uplus \; .... \; \uplus E^{\mathrm{T}}{}_n)$	M
	|	$\uplus E^{\mathrm{T}}{}_1 .. E^{\mathrm{T}}{}_n$	M
	|	$E^{\mathrm{T}} \setminus id_1 .. id_n$	M
	|	$(E^{\mathrm{T}}{}_1 \cap \; .... \; \cap E^{\mathrm{T}}{}_n)$	M
	|	$\cap E^{\mathrm{T}}{}_1 .. E^{\mathrm{T}}{}_n$	M
ts	::=		
	|	$t_1, .., t_n$	
E	::=		Definition environment and lexical environment
	|	$\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle$	
	|	ϵ	M
	|	$E \uplus E'$	
I	::=		Information given by type checking an expression
	|	$\langle \Sigma^{\mathrm{N}}, effect \rangle$	
	|	I_ϵ	Empty constraints, effect

	$I_1 \uplus I_2$	
	$I_1 \uplus .. \uplus I_n$	Unions the constraints and effect
formula	::=	
	judgement	
	$formula_1 \quad .. \quad formula_n$	
	$E^{\mathrm{K}}(tid) \triangleright kinf$	Kind lookup
	$E^{\mathrm{A}}(tid) \triangleright tinf$	
	$E^{\mathrm{A}}(tid) \triangleright ne$	
	$E^{\mathrm{T}}(id) \triangleright tinf$	Type lookup
	$E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\ tinf : tinf_1 ... tinf_n$	Overloaded type lookup
	$E^{\mathrm{K}}(tid) < -\vert k$	Update the kind associated with id to k
	$E^{\mathrm{R}}(id_0 .. id_n) \triangleright t, ts$	Record lookup
	$E^{\mathrm{R}}(t) \triangleright id_0 : t_0 .. id_n : t_n$	Record looup by type
	$E^{\mathrm{E}}(t) \triangleright enumerate\_map$	Enumeration lookup by type
	$\mathbf{dom}\,(E^{\mathrm{T}}{}_1) \cap \mathbf{dom}\,(E^{\mathrm{T}}{}_2) = \emptyset$	
	$\mathbf{dom}\,(E^{\mathrm{K}}{}_1) \cap \mathbf{dom}\,(E^{\mathrm{K}}{}_2) = \emptyset$	
	$\mathbf{disjoint\,doms}\,(E^{\mathrm{T}}{}_1, ..., E^{\mathrm{T}}{}_n)$	Pairwise disjoint domains
	$id \notin \mathbf{dom}\,(E^{\mathrm{K}})$	
	$id \notin \mathbf{dom}\,(E^{\mathrm{T}})$	
	$id_0 : t_0 .. id_n : t_n \subset id'_0 : t'_0 .. id'_i : t'_i$	
	$num_1 < ... < num_n$	
	$num_1 > ... > num_n$	
	$exp_1 \equiv exp_2$	
	$E^{\mathrm{K}}{}_1 \equiv E^{\mathrm{K}}{}_2$	
	$E^{\mathrm{K}}{}_1 \approx E^{\mathrm{K}}{}_2$	
	$E^{\mathrm{T}}{}_1 \equiv E^{\mathrm{T}}{}_2$	
	$E^{\mathrm{R}}_1 \equiv E^{\mathrm{R}}_2$	
	$E^{\mathrm{E}}_1 \equiv E^{\mathrm{E}}_2$	
	$E^{\mathrm{D}}{}_1 \equiv E^{\mathrm{D}}{}_2$	

| $E_1 \equiv E_2$
| $\Sigma^{\mathrm{N}}{}_1 \equiv \Sigma^{\mathrm{N}}{}_2$
| $id \equiv' id$
| $x_1 \neq x_2$
| $lit_1 \neq lit_2$
| $I_1 \equiv I_2$
| $effect_1 \equiv effect_2$
| $t_1 \equiv t_2$
| $ne \equiv ne'$
| $kid \equiv fresh\_kid(E^{\mathrm{D}})$

3.2 Type relations

$\boxed{E^{\mathrm{K}} \vdash_t t\,\mathbf{ok}}$ Well-formed types

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_Typ}{E^{\mathrm{K}} \vdash_t 'x\,\mathbf{ok}}\ \textsc{check\_t\_var}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_infer \quad E^{\mathrm{K}}('x) < -|K\_Typ}{E^{\mathrm{K}} \vdash_t 'x\,\mathbf{ok}}\ \textsc{check\_t\_varInfer}$$

$$\frac{E^{\mathrm{K}} \vdash_t t_1\,\mathbf{ok} \quad E^{\mathrm{K}} \vdash_t t_2\,\mathbf{ok} \quad E^{\mathrm{K}} \vdash_e effect\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_t t_1 \to t_2\ effect\,\mathbf{ok}}\ \textsc{check\_t\_fn}$$

$$\frac{E^{\mathrm{K}} \vdash_t t_1\,\mathbf{ok} \quad .... \quad E^{\mathrm{K}} \vdash_t t_n\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_t (t_1, ..., t_n)\,\mathbf{ok}}\ \textsc{check\_t\_tup}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 .. k_n \to K\_Typ) \quad E^{\mathrm{K}}, k_1 \vdash t\_arg_1\,\mathbf{ok} \quad .. \quad E^{\mathrm{K}}, k_n \vdash t\_arg_n\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_t x\langle t\_arg_1 .. t\_arg_n\rangle\,\mathbf{ok}}\ \textsc{check\_t\_app}$$

$\boxed{E^{\mathrm{K}} \vdash_e effect\,\mathbf{ok}}$ Well-formed effects

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_Efct}{E^{\mathrm{K}} \vdash_e 'x\ \mathbf{ok}}\ \textsc{check\_ef\_var}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_infer \quad E^{\mathrm{K}}('x) < -|K\_Efct}{E^{\mathrm{K}} \vdash_e 'x\ \mathbf{ok}}\ \textsc{check\_ef\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_e \{base\_effect_1, .., base\_effect_n\}\ \mathbf{ok}}\ \textsc{check\_ef\_set}$$

$\boxed{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n x\ \mathbf{ok}}\ \textsc{check\_n\_id}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n 'x\ \mathbf{ok}}\ \textsc{check\_n\_var}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_infer \quad E^{\mathrm{K}}('x) < -|K\_Nat}{E^{\mathrm{K}} \vdash_n 'x\ \mathbf{ok}}\ \textsc{check\_n\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_n num\ \mathbf{ok}}\ \textsc{check\_n\_num}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \quad E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n ne_1 + ne_2\ \mathbf{ok}}\ \textsc{check\_n\_sum}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \quad E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n ne_1 - ne_2\ \mathbf{ok}}\ \textsc{check\_n\_minus}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \quad E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n ne_1 * ne_2\ \mathbf{ok}}\ \textsc{check\_n\_mult}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n 2 ** ne\ \mathbf{ok}}\ \textsc{check\_n\_exp}$$

$\boxed{E^{\mathrm{K}} \vdash_o order\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_Ord}{E^{\mathrm{K}} \vdash_o 'x\ \mathbf{ok}}\ \textsc{check\_ord\_var}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_infer \quad E^{\mathrm{K}}('x) < -|K\_Ord}{E^{\mathrm{K}} \vdash_o 'x\ \mathbf{ok}}\ \textsc{check\_ord\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_o \mathbf{inc}\ \mathbf{ok}}\ \textsc{check\_ord\_inc}$$

$$\frac{}{E^{\mathrm{K}} \vdash_o \mathbf{dec}\ \mathbf{ok}}\ \textsc{check\_ord\_dec}$$

$\boxed{E^{\mathrm{K}}, k \vdash t\_arg\ \mathbf{ok}}$ Well-formed type arguments kind check matching the application type variable

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Typ \vdash t\ \mathbf{ok}}\ \textsc{check\_targs\_typ}$$

$$\frac{E^{\mathrm{K}} \vdash_e effect\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Efct \vdash effect\ \mathbf{ok}}\ \textsc{check\_targs\_eff}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Nat \vdash ne\ \mathbf{ok}}\ \textsc{check\_targs\_nat}$$

$$\frac{E^{\mathrm{K}} \vdash_o order\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Ord \vdash order\ \mathbf{ok}}\ \textsc{check\_targs\_ord}$$

$\boxed{E^{\mathrm{K}} \vdash kind \rightsquigarrow k}$

$$\frac{}{E^{\mathrm{K}} \vdash \mathbf{Type} \rightsquigarrow K\_Typ}\ \textsc{convert\_kind\_Typ}$$

$\boxed{E^{\mathrm{D}} \vdash quant\_item \rightsquigarrow {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}$ Convert source quantifiers to kind environments and constraints

$$\frac{E^{\mathrm{K}} \vdash kind \rightsquigarrow k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash kind\ 'x \rightsquigarrow \{'x \mapsto k\}, \{\,\}}\ \textsc{convert\_quants\_kind}$$

$$\frac{E^{\mathrm{K}}('x) \triangleright k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash 'x \rightsquigarrow \{'x \mapsto k\}, \{\,\}} \quad \textsc{convert\_quants\_nokind}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 = nexp_2 \rightsquigarrow \{\,\}, \{ne_1 = ne_2\}} \quad \textsc{convert\_quants\_eq}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 \geq nexp_2 \rightsquigarrow \{\,\}, \{ne_1 \geq ne_2\}} \quad \textsc{convert\_quants\_gteq}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 \leq nexp_2 \rightsquigarrow \{\,\}, \{ne_1 \leq ne_2\}} \quad \textsc{convert\_quants\_lteq}$$

$$\frac{}{E^{\mathrm{D}} \vdash 'x\,\mathbf{IN}\,\{num_1, \ldots, num_n\} \rightsquigarrow \{\,\}, \{'x\,\mathbf{IN}\,\{num_1, \ldots, num_n\}\}} \quad \textsc{convert\_quants\_in}$$

$\boxed{E^{\mathrm{D}} \vdash typschm \rightsquigarrow t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}$ Convert source types with typeschemes to internal types and kind environments

$$\frac{E^{\mathrm{D}} \vdash typ \rightsquigarrow t}{E^{\mathrm{D}} \vdash\ typ \rightsquigarrow t, \{\,\}, \{\,\}} \quad \textsc{convert\_typschm\_noquant}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash quant\_item_1 \rightsquigarrow {E^{\mathrm{K}}}_1, {\Sigma^{\mathrm{N}}}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash quant\_item_n \rightsquigarrow {E^{\mathrm{K}}}_n, {\Sigma^{\mathrm{N}}}_n \\ E^{\mathrm{K}} \equiv {E^{\mathrm{K}}}_1 \uplus \ldots \uplus {E^{\mathrm{K}}}_n \\ E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}, \{\,\}, \{\,\}, \{\,\}\rangle \vdash typ \rightsquigarrow t\end{array}}{E^{\mathrm{D}} \vdash \mathbf{forall}\, quant\_item_1, \ldots, quant\_item_n.\, typ \rightsquigarrow t, E^{\mathrm{K}}, {\Sigma^{\mathrm{N}}}_1 \uplus \ldots \uplus {\Sigma^{\mathrm{N}}}_n} \quad \textsc{convert\_typschm\_quant}$$

$\boxed{E^{\mathrm{D}} \vdash typ \rightsquigarrow t}$ Convert source types to internal types

$$\frac{E^{\mathrm{K}}('x) \triangleright K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash 'x \rightsquigarrow 'x} \quad \textsc{convert\_typ\_var}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash x \rightsquigarrow x} \quad \textsc{convert\_typ\_id}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}} \vdash typ_1 \rightsquigarrow t_1 \\ E^{\mathrm{D}} \vdash typ_2 \rightsquigarrow t_2\end{array}}{E^{\mathrm{D}} \vdash typ_1 \to typ_2\,\mathbf{effect}\,effect \rightsquigarrow t_1 \to t_2\,effect} \quad \textsc{convert\_typ\_fn}$$

$$\frac{E^{\mathrm{D}} \vdash typ_1 \rightsquigarrow t_1 \quad .... \quad E^{\mathrm{D}} \vdash typ_n \rightsquigarrow t_n}{E^{\mathrm{D}} \vdash (typ_1, \, ... \, , typ_n) \rightsquigarrow (t_1, \, ... \, , t_n)} \quad \text{CONVERT\_TYP\_TUP}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \, .. \, k_n \rightarrow K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, k_1 \vdash typ\_arg_1 \rightsquigarrow t\_arg_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, k_n \vdash typ\_arg_n \rightsquigarrow t\_arg_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash x \langle typ\_arg_1, \, .. \, , typ\_arg_n \rangle \rightsquigarrow x \langle t\_arg_1 \, .. \, t\_arg_n \rangle} \quad \text{CONVERT\_TYP\_APP}$$

$\boxed{E^{\mathrm{D}}, k \vdash typ\_arg \rightsquigarrow t\_arg}$ Convert source type arguments to internals

$$\frac{E^{\mathrm{D}} \vdash typ \rightsquigarrow t}{E^{\mathrm{D}}, K\_Typ \vdash typ \rightsquigarrow t} \quad \text{CONVERT\_TARG\_TYP}$$

$\boxed{\vdash nexp \rightsquigarrow ne}$ Convert and normalize numeric expressions

$$\frac{}{\vdash x \rightsquigarrow x} \quad \text{CONVERT\_NEXP\_ID}$$

$$\frac{}{\vdash 'x \rightsquigarrow 'x} \quad \text{CONVERT\_NEXP\_VAR}$$

$$\frac{}{\vdash num \rightsquigarrow num} \quad \text{CONVERT\_NEXP\_NUM}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2 \end{array}}{\vdash nexp_1 * nexp_2 \rightsquigarrow ne_1 * ne_2} \quad \text{CONVERT\_NEXP\_MULT}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2 \end{array}}{\vdash nexp_1 + nexp_2 \rightsquigarrow ne_1 + ne_2} \quad \text{CONVERT\_NEXP\_ADD}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \rightsquigarrow ne_1 \\ \vdash nexp_2 \rightsquigarrow ne_2 \end{array}}{\vdash nexp_1 - nexp_2 \rightsquigarrow ne_1 - ne_2} \quad \text{CONVERT\_NEXP\_SUB}$$

$$\frac{\vdash nexp \rightsquigarrow ne}{\vdash 2 ** nexp \rightsquigarrow 2 ** ne} \quad \text{CONVERT\_NEXP\_EXP}$$

$\boxed{E^{\mathrm{D}} \vdash_n ne \approx ne'}$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash_n ne \approx ne}\ \textsc{conforms\_to\_ne\_refl}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_1 \approx ne_2 \quad E^{\mathrm{D}} \vdash_n ne_2 \approx ne_3}{E^{\mathrm{D}} \vdash_n ne_1 \approx ne_3}\ \textsc{conforms\_to\_ne\_trans}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx ne_1}{E^{\mathrm{D}} \vdash_n ne_1 \approx ne_2}\ \textsc{conforms\_to\_ne\_assoc}$$

$$\frac{E^{\mathrm{A}}(x) \triangleright ne \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash_n ne \approx ne'}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash_n x \approx ne'}\ \textsc{conforms\_to\_ne\_abbrev}$$

$$\frac{num \equiv num'}{E^{\mathrm{D}} \vdash_n num \approx num'}\ \textsc{conforms\_to\_ne\_constants}$$

$$\frac{}{E^{\mathrm{D}} \vdash_n ne \approx ne'}\ \textsc{conforms\_to\_ne\_rest}$$

$\boxed{E^{\mathrm{D}} \vdash t \approx t'}$ Relate t and t' when t can be used where t' is expected without consideration for non-constant nats

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash t \approx t}\ \textsc{conforms\_to\_refl}$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \approx t_2 \quad E^{\mathrm{D}} \vdash t_2 \approx t_3}{E^{\mathrm{D}} \vdash t_1 \approx t_3}\ \textsc{conforms\_to\_trans}$$

$$\frac{}{E^{\mathrm{D}} \vdash {'x} \approx t}\ \textsc{conforms\_to\_var}$$

$$\frac{}{E^{\mathrm{D}} \vdash t \approx {'x}}\ \textsc{conforms\_to\_var2}$$

$$\frac{E^{\mathrm{A}}(x) \triangleright u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash u \approx t}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash x \approx t}\ \textsc{conforms\_to\_abbrev}$$

$$\frac{E^{\mathrm{A}}(x) \triangleright u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx u}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx x}\ \textsc{conforms\_to\_abbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash t_n \approx u_n}{E^{\mathrm{D}} \vdash (t_1, \ldots, t_n) \approx (u_1, \ldots, u_n)}\ \textsc{conforms\_to\_tup}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 .. k_n \to K\_Typ) \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash t\_arg_1 \approx t\_arg'_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash t\_arg_n \approx t\_arg'_n}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 .. t\_arg_n\rangle \approx x\langle t\_arg'_1 .. t\_arg'_n\rangle}\ \textsc{conforms\_to\_app}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{atom}\,\langle ne\rangle \approx \mathbf{range}\,\langle ne_1\ ne_2\rangle}\ \textsc{conforms\_to\_atom}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \approx \mathbf{atom}\,\langle ne\rangle}\ \textsc{conforms\_to\_atom2}$$

$$\frac{x' \neq x \quad E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, .., tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 .. t\_arg_n\rangle \approx u[t\_arg'_1/tid_1 .. t\_arg'_m/tid_m]}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 .. t\_arg_n\rangle \approx x'\langle t\_arg'_1 .. t\_arg'_m\rangle}\ \textsc{conforms\_to\_appAbbrev}$$

$$\frac{x' \neq x \quad E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u[t\_arg_1/tid_1 .. t\_arg_n/tid_n] \approx x\langle t\_arg'_1 .. t\_arg'_m\rangle}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x'\langle t\_arg_1 .. t\_arg_n\rangle \approx x\langle t\_arg'_1 .. t\_arg'_m\rangle}\ \textsc{conforms\_to\_appAbbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{register}\,\langle t\rangle \approx u}\ \textsc{conforms\_to\_register}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{reg}\,\langle t\rangle \approx u}\ \textsc{conforms\_to\_reg}$$

$\boxed{E^{\mathrm{D}}, k \vdash t\_arg \approx t\_arg'}$

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash t \approx t'}\ \textsc{targconforms\_typ}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne \approx ne'}{E^{\mathrm{D}}, K\_Nat \vdash ne \approx ne'} \quad \textsc{targconforms\_nexp}$$

$\boxed{E^{\mathrm{D}} \vdash_c t \approx t'}$ Relate t and t' when t can be used where t' is expected upto applying coercions to t

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}} \vdash_c t \approx t'} \quad \textsc{conforms\_to\_upto\_coerce\_base}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx \mathbf{one}}{E^{\mathrm{D}} \vdash_c \mathbf{bit} \approx \mathbf{vector}\,\langle ne\ ne_2\ order\ \mathbf{bit}\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_bit}\textsc{ToVec}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx \mathbf{one}}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\ ne_2\ order\ \mathbf{bit}\rangle \approx \mathbf{bit}} \quad \textsc{conforms\_to\_upto\_coerce\_vec}\textsc{ToBit}$$

$$\frac{}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\ ne_2\ order\ \mathbf{bit}\rangle \approx \mathbf{atom}\,\langle ne_3\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_vec}\textsc{ToAtom}$$

$$\frac{}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\ ne_2\ order\ \mathbf{bit}\rangle \approx \mathbf{range}\,\langle ne_3\ ne_4\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_vec}\textsc{ToRange}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright\ enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x \approx \mathbf{range}\,\langle ne_1\ ne_2\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_enum}\textsc{ToRange}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright\ enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c \mathbf{range}\,\langle ne_1\ ne_2\rangle \approx x} \quad \textsc{conforms\_to\_upto\_coerce\_range}\textsc{ToEnum}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright\ enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x \approx \mathbf{atom}\,\langle ne\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_enum}\textsc{ToAtom}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright\ enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c \mathbf{atom}\,\langle ne\rangle \approx x} \quad \textsc{conforms\_to\_upto\_coerce\_atom}\textsc{ToEnum}$$

$$\frac{E^{\mathrm{D}} \vdash_c t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash_c t_n \approx u_n}{E^{\mathrm{D}} \vdash_c (t_1,\ ....,\ t_n) \approx (u_1,\ ....,\ u_n)} \quad \textsc{conforms\_to\_upto\_coerce\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright\ K\_Lam(k_1\,..\,k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash_c t\_arg_1 \approx t\_arg_1' \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash_c t\_arg_n \approx t\_arg_n' \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1\,..\,t\_arg_n\rangle \approx x\langle t\_arg_1'\,..\,t\_arg_n'\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\ ..,\ tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1\ ..\ t\_arg_n\rangle \approx u[t\_arg'_1/tid_1\ ..\ t\_arg'_m/tid_m] \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1\ ..\ t\_arg_n\rangle \approx x'\langle t\_arg'_1\ ..\ t\_arg'_m\rangle}\ \textsc{conforms\_to\_upto\_coerce\_appAbbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\ ..,\ tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c u[t\_arg_1/tid_1\ ..\ t\_arg_n/tid_n] \approx x\langle t\_arg'_1\ ..\ t\_arg'_m\rangle \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x'\langle t\_arg_1\ ..\ t\_arg_n\rangle \approx x\langle t\_arg'_1\ ..\ t\_arg'_m\rangle}\ \textsc{conforms\_to\_upto\_coerce\_appAbbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx u}{E^{\mathrm{D}} \vdash_c \mathbf{register}\ \langle t\rangle \approx u}\ \textsc{conforms\_to\_upto\_coerce\_register}$$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx u}{E^{\mathrm{D}} \vdash_c \mathbf{reg}\ \langle t\rangle \approx u}\ \textsc{conforms\_to\_upto\_coerce\_reg}$$

$\boxed{E^{\mathrm{D}}, k \vdash_c t\_arg \approx t\_arg'}$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash_c t \approx t'}\ \textsc{targconforms\_coerce\_typ}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne \approx ne'}{E^{\mathrm{D}}, K\_Nat \vdash_c ne \approx ne'}\ \textsc{targconforms\_coerce\_nexp}$$

$\boxed{\sigma_{conformsto(t,t')}(tinflist) \triangleright tinflist'}$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ E^{\mathrm{D}} \vdash t'_j \approx t_j \\ \sigma_{\mathbf{full}(t_i,t_j)}(tinf_0\ ..\ tinf_m\ tinf'_0\ ..\ tinf'_n) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{full}(t_i,t_j)}(tinf_0\ ..\ tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j\ effect\ tinf'_0\ ..\ tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j}\ \textsc{so\_full}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ \sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0\ ..\ tinf_m) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0\ ..\ tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t\ effect\ tinf'_0\ ..\ tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t}\ \textsc{so\_parm}$$

$\boxed{E^{\mathrm{D}}, widening \vdash t \precapprox t' : t'', \Sigma^{\mathrm{N}}}$ t is consistent with t' if they match if t can be used where t' is needed after the constraints are solved, with no coercions needed. t" is t

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \mathit{widening} \vdash t \precsim t : t, \{\,\}} \quad \textsc{consistent\_typ\_refl}$$

$$\frac{E^{\mathrm{D}}, \mathit{widening} \vdash t_1 \precsim t_3 : t_4, {\Sigma^{\mathrm{N}}}_1 \quad E^{\mathrm{D}}, \mathit{widening} \vdash t_4 \precsim t_2 : t_5, {\Sigma^{\mathrm{N}}}_2}{E^{\mathrm{D}}, \mathit{widening} \vdash t_1 \precsim t_2 : t_5, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2} \quad \textsc{consistent\_typ\_trans}$$

$$\frac{E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \mathit{tag}, u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \mathit{widening} \vdash u \precsim t : t', \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \mathit{widening} \vdash x \precsim t : t', \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \textsc{consistent\_typ\_abbrev}$$

$$\frac{E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \mathit{tag}, u \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \mathit{widening} \vdash t \precsim u : t', \Sigma^{\mathrm{N}\prime}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \mathit{widening} \vdash t \precsim x : t', \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \textsc{consistent\_typ\_abbrev}2$$

$$\frac{}{E^{\mathrm{D}}, \mathit{widening} \vdash {'x} \precsim t : t, \{\,\}} \quad \textsc{consistent\_typ\_var}$$

$$\frac{}{E^{\mathrm{D}}, \mathit{widening} \vdash t \precsim {'x} : t, \{\,\}} \quad \textsc{consistent\_typ\_var}2$$

$$\frac{E^{\mathrm{D}}, \mathit{widening} \vdash t_1 \precsim u_1 : u_1', {\Sigma^{\mathrm{N}}}_1 \quad \ldots. \quad E^{\mathrm{D}}, \mathit{widening} \vdash t_n \precsim u_n : u_n', {\Sigma^{\mathrm{N}}}_n}{E^{\mathrm{D}}, \mathit{widening} \vdash (t_1, \ldots., t_n) \precsim (u_1, \ldots., u_n) : (u_1', \ldots., u_n'), {\Sigma^{\mathrm{N}}}_1 \uplus \ldots. \uplus {\Sigma^{\mathrm{N}}}_n} \quad \textsc{consistent\_typ\_tup}$$

$$\frac{}{E^{\mathrm{D}}, \mathit{widening} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \precsim \mathbf{range}\,\langle ne_3\ ne_4\rangle : \mathbf{range}\,\langle ne_3\ ne_4\rangle, \{ne_3 \leq ne_1, ne_2 \leq ne_4\}} \quad \textsc{consistent\_typ\_range}$$

$$\frac{}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle ne\rangle \precsim \mathbf{range}\,\langle ne_1\ ne_2\rangle : \mathbf{atom}\,\langle ne\rangle, \{ne_1 \leq ne, ne \leq ne_2\}} \quad \textsc{consistent\_typ\_atomRange}$$

$$\frac{}{E^{\mathrm{D}}, (\mathbf{none}, \_) \vdash \mathbf{atom}\,\langle ne_1\rangle \precsim \mathbf{atom}\,\langle ne_2\rangle : \mathbf{atom}\,\langle ne_2\rangle, \{ne_1 = ne_2\}} \quad \textsc{consistent\_typ\_atom}$$

$$\frac{num_1 \ < \ num_2}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle num_1\rangle \precsim \mathbf{atom}\,\langle num_2\rangle : \mathbf{range}\,\langle num_1\ num_2\rangle, \{\,\}} \quad \textsc{consistent\_typ\_atomWidenConst}$$

$$\frac{num_2 \ < \ num_1}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle num_1\rangle \precsim \mathbf{atom}\,\langle num_2\rangle : \mathbf{range}\,\langle num_2\ num_1\rangle, \{\,\}} \quad \textsc{consistent\_typ\_atomWidenConst}2$$

$$\frac{}{E^{\mathrm{D}}, \mathit{widening} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \precsim \mathbf{atom}\,\langle {'x}\rangle : \mathbf{atom}\,\langle {'x}\rangle, \{ne_1 \leq {'x}, {'x} \leq ne_2\}} \quad \textsc{consistent\_typ\_rangeAtom}$$

$$\frac{E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash t \precsim t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, (\_, \mathbf{none}) \vdash \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle \precsim \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t'\rangle : \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t''\rangle, \{ne_2 = ne_4, ne_1 = ne_3\} \uplus \Sigma^{\mathrm{N}}}\ \textsc{consistent\_typ\_vector}$$

$$\frac{E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash t \precsim t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, (\_, \mathbf{vectors}) \vdash \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle \precsim \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t'\rangle : \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t''\rangle, \{ne_2 = ne_4\} \uplus \Sigma^{\mathrm{N}}}\ \textsc{consistent\_typ\_vectorWiden}$$

$$\frac{\begin{array}{l}E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1\ ..\ k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, k_1 \vdash t\_arg_1 \precsim t\_arg'_1, {\Sigma^{\mathrm{N}}}_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, k_n \vdash t\_arg_n \precsim t\_arg'_n, {\Sigma^{\mathrm{N}}}_n\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1\ ..\ t\_arg_n\rangle \precsim x\langle t\_arg'_1\ ..\ t\_arg'_n\rangle : x\langle t\_arg'_1\ ..\ t\_arg'_n\rangle, {\Sigma^{\mathrm{N}}}_1 \uplus\ ..\ \uplus {\Sigma^{\mathrm{N}}}_n}\ \textsc{consistent\_typ\_app}$$

$$\frac{\begin{array}{l}x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\ ..\ , tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1\ ..\ t\_arg_n\rangle \precsim u[t\_arg'_1/tid_1\ ..\ t\_arg'_m/tid_m] : t, {\Sigma^{\mathrm{N}}}_2\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1\ ..\ t\_arg_n\rangle \precsim x'\langle t\_arg'_1\ ..\ t\_arg'_m\rangle : x'\langle t\_arg'_1\ ..\ t\_arg'_m\rangle, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_2}\ \textsc{consistent\_typ\_appAbbrev}$$

$$\frac{\begin{array}{l}x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\ ..\ , tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u[t\_arg'_1/tid_1\ ..\ t\_arg'_m/tid_m] \precsim x\langle t\_arg_1\ ..\ t\_arg_n\rangle : t, {\Sigma^{\mathrm{N}}}_2\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x'\langle t\_arg'_1\ ..\ t\_arg'_m\rangle \precsim x\langle t\_arg_1\ ..\ t\_arg_n\rangle : x\langle t\_arg_1\ ..\ t\_arg_n\rangle, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_2}\ \textsc{consistent\_typ\_appAbbrev2}$$

$\boxed{E^{\mathrm{D}}, widening, k \vdash t\_arg \precsim t\_arg', \Sigma^{\mathrm{N}}}$

$$\frac{E^{\mathrm{D}}, widening \vdash t \precsim t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, widening, K\_Typ \vdash t \precsim t', \Sigma^{\mathrm{N}}}\ \textsc{targ\_consistent\_typ}$$

$$\frac{}{E^{\mathrm{D}}, widening, K\_Nat \vdash ne \precsim ne', \{ne = ne'\}}\ \textsc{targ\_consistent\_nexp}$$

$\boxed{E^{\mathrm{D}}, widening, t' \vdash exp : t \triangleright t'', exp', \Sigma^{\mathrm{N}}, effect}$

$$\frac{\begin{array}{l}E^{\mathrm{D}}, widening, u_1 \vdash id_1 : t_1 \triangleright u_1, exp_1, {\Sigma^{\mathrm{N}}}_1, effect_1 \quad .. \quad E^{\mathrm{D}}, widening, u_n \vdash id_n : t_n \triangleright u_n, exp_n, {\Sigma^{\mathrm{N}}}_n, effect_n \\ exp' \equiv \mathbf{switch}\ exp\{\ \mathbf{case}\ (id_1,\ ..\ , id_n) \to (exp_1,\ ..\ , exp_n)\}\end{array}}{E^{\mathrm{D}}, widening, (u_1,\ ..\ , u_n) \vdash exp : (t_1,\ ..\ , t_n) \triangleright (u_1,\ ..\ , u_n), exp', {\Sigma^{\mathrm{N}}}_1 \uplus\ ..\ \uplus {\Sigma^{\mathrm{N}}}_n, \mathbf{pure}}\ \textsc{coerce\_typ\_tuple}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u \precsim t : t', \Sigma^{\mathrm{N}} \\ exp' \equiv (annot)exp\end{array}}{E^{\mathrm{D}}, widening, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\ order\ u\rangle \triangleright \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t'\rangle, exp', \Sigma^{\mathrm{N}} \uplus \{ne_2 = ne_4\}, \mathbf{pure}}\ \textsc{coerce\_typ\_vectorUpdateStart}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}}, (\mathbf{none}, \mathbf{none}) \vdash u \precsim\!\approx \mathbf{bit} : \mathbf{bit}, \Sigma^{\mathrm{N}} \\ exp' \equiv to\_num\ exp\end{array}}{E^{\mathrm{D}}, widening, \mathbf{range}\ \langle ne_1\ ne_2\rangle \vdash exp : \mathbf{vector}\ \langle ne_3\ ne_4\ order\ u\rangle \triangleright \mathbf{range}\ \langle ne_1\ ne_2\rangle, exp', \Sigma^{\mathrm{N}} \uplus \{ne_1 = \mathbf{zero}, ne_2 \geq 2 ** ne_4\}, \mathbf{pure}}\ \textsc{coerce\_typ\_toNum}$$

$$\frac{exp' \equiv to\_vec\ exp}{E^{\mathrm{D}}, widening, \mathbf{vector}\ \langle ne_1\ ne_2\ order\ \mathbf{bit}\rangle \vdash exp : \mathbf{range}\ \langle ne_3\ ne_4\rangle \triangleright \mathbf{vector}\ \langle ne_1\ ne_2\ order\ \mathbf{bit}\rangle, exp', \{ne_3 = \mathbf{zero}, ne_4 \leq 2 ** ne_2\}, \mathbf{pure}}\ \textsc{coerce\_typ\_fromNum}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}} \vdash typ \rightsquigarrow t \\ exp' \equiv (typ)exp \\ E^{\mathrm{D}}, widening, u \vdash exp' : t \triangleright t', exp'', \Sigma^{\mathrm{N}}, \mathbf{pure}\end{array}}{E^{\mathrm{D}}, widening, u \vdash exp : \mathbf{register}\ \langle t\rangle \triangleright t', exp'', \Sigma^{\mathrm{N}}, \{\mathbf{rreg}\}}\ \textsc{coerce\_typ\_readReg}$$

$$\frac{exp' \equiv \mathbf{msb}\ (exp)}{E^{\mathrm{D}}, widening, \mathbf{bit} \vdash exp : \mathbf{vector}\ \langle ne_1\ ne_2\ order\ \mathbf{bit}\rangle \triangleright \mathbf{bit}, exp', \{ne_1 = \mathbf{one}\}, \mathbf{pure}}\ \textsc{coerce\_typ\_accessVecBit}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}}, widening \vdash \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one}\rangle \precsim\!\approx \mathbf{range}\ \langle ne_1\ ne_2\rangle : t, \Sigma^{\mathrm{N}} \\ exp' \equiv \mathbf{switch}\ exp\{\ \mathbf{case}\ \mathbf{bitzero} \rightarrow numZero\ \mathbf{case}\ \mathbf{bitone} \rightarrow numOne\}\end{array}}{E^{\mathrm{D}}, widening, \mathbf{range}\ \langle ne_1\ ne_2\rangle \vdash exp : \mathbf{bit} \triangleright \mathbf{range}\ \langle ne_1\ ne_2\rangle, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}}\ \textsc{coerce\_typ\_bitToNum}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}}, widening \vdash \mathbf{range}\ \langle ne_1\ ne_2\rangle \precsim\!\approx \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one}\rangle : t, \Sigma^{\mathrm{N}} \\ exp' \equiv \mathbf{switch}\ exp\{\ \mathbf{case}\ numZero \rightarrow \mathbf{bitzero}\ \mathbf{case}\ numOne \rightarrow \mathbf{bitone}\}\end{array}}{E^{\mathrm{D}}, widening, \mathbf{bit} \vdash exp : \mathbf{range}\ \langle ne_1\ ne_2\rangle \triangleright \mathbf{bit}, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}}\ \textsc{coerce\_typ\_numToBit}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash \mathbf{atom}\ \langle ne\rangle \precsim\!\approx \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one}\rangle : t, \Sigma^{\mathrm{N}} \\ exp' \equiv \mathbf{switch}\ exp\{\ \mathbf{case}\ numZero \rightarrow \mathbf{bitzero}\ \mathbf{case}\ numOne \rightarrow \mathbf{bitone}\}\end{array}}{E^{\mathrm{D}}, widening, \mathbf{bit} \vdash exp : \mathbf{atom}\ \langle ne\rangle \triangleright \mathbf{bit}, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}}\ \textsc{coerce\_typ\_numToBitAtom}$$

$$\frac{\begin{array}{c}E^{\mathrm{E}}(x) \triangleright \{\ \overline{num_i \mapsto id_i}^{\,i}\ \} \\ exp' \equiv \mathbf{switch}\ exp\{\ \overline{\mathbf{case}\ num_i \rightarrow id_i}^{\,i}\ \} \\ ne_3 \equiv \mathbf{count}\ (\ \overline{num_i}^{\,i}\ )\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, x \vdash exp : \mathbf{range}\ \langle ne_1\ ne_2\rangle \triangleright x, exp', \{ne_1 \leq \mathbf{zero}, ne_2 \leq ne_3\}, \mathbf{pure}}\ \textsc{coerce\_typ\_toEnumerate}$$

$$\frac{\begin{array}{c}E^{\mathrm{E}}(x) \triangleright \{\ \overline{num_i \mapsto id_i}^{\,i}\ \} \\ exp' \equiv \mathbf{switch}\ exp\{\ \overline{\mathbf{case}\ id_i \rightarrow num_i}^{\,i}\ \} \\ ne_3 \equiv \mathbf{count}\ (\ \overline{num_i}^{\,i}\ ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, (\mathbf{nums}, \mathbf{none}) \vdash \mathbf{range}\ \langle \mathbf{zero}\ ne_3\rangle \precsim\!\approx \mathbf{range}\ \langle ne_1\ ne_2\rangle : t, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, \mathbf{range}\ \langle ne_1\ ne_2\rangle \vdash exp : x \triangleright \mathbf{range}\ \langle \mathbf{zero}\ ne_3\rangle, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}}\ \textsc{coerce\_typ\_fromEnumerate}$$

$$\frac{E^{\mathrm{D}}, \mathit{widening} \vdash t \precsim u : u', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, \mathit{widening}, u \vdash \mathit{exp} : t \triangleright u', \mathit{exp}, \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \text{COERCE\_TYP\_EQ}$$

$\boxed{\mathit{widening}, t \vdash \mathit{lit} : t' \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}$ Typing literal constants, coercing to expected type t

$$\frac{}{\mathit{widening}, \mathbf{range}\,\langle \mathit{ne}\ \mathit{ne}'\rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num}\rangle \Rightarrow \mathit{num}, \{\mathit{ne} \leq \mathit{num}, \mathit{num} \leq \mathit{ne}'\}} \quad \text{CHECK\_LIT\_NUM}$$

$$\frac{}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}\ \mathit{ne}'\ \mathit{order}\,\mathbf{bit}\rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num}\rangle \Rightarrow \mathit{to\_vec}\,\mathit{num}, \{\mathit{num} + \mathbf{one} \leq 2 ** \mathit{ne}'\}} \quad \text{CHECK\_LIT\_NUMTOVEC}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathit{numZero} : \mathbf{atom}\,\langle \mathbf{zero}\rangle \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITZERO}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathit{numOne} : \mathbf{atom}\,\langle \mathbf{one}\rangle \Rightarrow \mathbf{bitone}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITONE}$$

$$\frac{}{\mathit{widening}, \mathit{string} \vdash \mathit{string} : \mathit{string} \Rightarrow \mathit{string}, \{\,\}} \quad \text{CHECK\_LIT\_STRING}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{hex})}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\,\mathbf{bit}\rangle \vdash \mathit{hex} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\,\mathbf{bit}\rangle \Rightarrow \mathit{hex}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_HEX}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{bin})}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\,\mathbf{bit}\rangle \vdash \mathit{bin} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\,\mathbf{bit}\rangle \Rightarrow \mathit{bin}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_BIN}$$

$$\frac{}{\mathit{widening}, \mathbf{unit} \vdash () : \mathbf{unit} \Rightarrow \mathbf{unit}, \{\,\}} \quad \text{CHECK\_LIT\_UNIT}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathbf{bitzero} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITZERO}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathbf{bitone} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITONE}$$

$$\frac{}{\mathit{widening}, t \vdash \mathbf{undefined} : t \Rightarrow \mathbf{undefined}, \{\,\}} \quad \text{CHECK\_LIT\_UNDEF}$$

$\boxed{E, t \vdash \mathit{pat} : t' \triangleright \mathit{pat}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Typing patterns, building their binding environment

$$\frac{\begin{array}{l}\mathit{lit} \neq \mathbf{undefined} \\ (\mathbf{none}, \mathbf{none}), t \vdash \mathit{lit} : u \Rightarrow \mathit{lit}', \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u \precsim t : t', \Sigma^{\mathrm{N}\prime}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{lit} : t' \triangleright \mathit{lit}', \{\,\}, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \text{CHECK\_PAT\_LIT}$$

$$\frac{}{E, t \vdash \_ : t \triangleright \_, \{\,\}, \{\,\}} \quad \textsc{check\_pat\_wild}$$

$$\frac{\begin{array}{l} E, t \vdash pat : u \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ id \notin \mathbf{dom}\,(E^{\mathrm{T}}{}_1) \end{array}}{E, t \vdash (pat\ \mathbf{as}\ id) : u \triangleright (pat'\ \mathbf{as}\ id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_as}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, \mathbf{Default}, t' \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t' \vdash pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}}, (\mathbf{none}, \mathbf{none}) \vdash t' \precapprox u : u', \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash (pat\ \mathbf{as}\ id) : t \triangleright (pat'\ \mathbf{as}\ id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t'\}), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \textsc{check\_pat\_asDefault}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash typ \rightsquigarrow t \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash (typ) pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_typ}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \ldots, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, (u'_1, \ldots, u'_n) \to x\langle t\_arg_1 \ldots t\_arg_m \rangle\ \mathbf{pure} \\ (u_1, \ldots, u_n) \to x\langle t\_args' \rangle\ \mathbf{pure} \equiv (u'_1, \ldots, u'_n) \to x\langle t\_args \rangle\ \mathbf{pure}[t\_arg_1/tid_1 \ldots t\_arg_m/tid_m] \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u_1 \vdash pat_1 : t_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u_n \vdash pat_n : t_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash x\langle t\_args' \rangle \precapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id(pat_1, \ldots, pat_n) : x\langle t\_args' \rangle \triangleright id(pat'_1, \ldots, pat'_n), \uplus E^{\mathrm{T}}{}_1 \ldots E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n} \quad \textsc{check\_pat\_constr}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \ldots, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, \mathbf{unit} \to x\langle t\_arg_1 \ldots t\_arg_m \rangle\ \mathbf{pure} \\ \mathbf{unit} \to x\langle t\_args' \rangle\ \mathbf{pure} \equiv \mathbf{unit} \to x\langle t\_args \rangle\ \mathbf{pure}[t\_arg_1/tid_1 \ldots t\_arg_m/tid_m] \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash x\langle t\_args' \rangle \precapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : t \triangleright id, \{\,\}, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_identConstr}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, \mathbf{Default}, t \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash t \precapprox u : u', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash id : t \triangleright id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_varDefault}$$

$$\frac{}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : t \triangleright id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \{\,\}} \quad \textsc{check\_pat\_var}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(\ \overline{id_i}^{\,i}\ ) \triangleright x\langle t\_args\rangle, (\ \overline{t_i}^{\,i}\ ) \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash pat_i : u_i \triangleright pat_i', E^{\mathrm{T}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \mathbf{disjoint\ doms}\ (\ \overline{E^{\mathrm{T}}{}_i}^{\,i}\ ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, (\mathbf{nums}, \mathbf{vectors}) \vdash x\langle t\_args\rangle \gtrapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\ \overline{id_i = pat_i}^{\,i}\ ;^{?}\} : x\langle t\_args\rangle \triangleright \{\ \overline{id_i = pat_i'}^{\,i}\ ;^{?}\}, \uplus\, \overline{E^{\mathrm{T}}{}_i}^{\,i}, \Sigma^{\mathrm{N}} \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}}\ \text{CHECK\_PAT\_RECORD}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad ... \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat_n', E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\ (E^{\mathrm{T}}{}_1, \ ..., E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_1 \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_1 \quad ... \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_n \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_n \\ ne_4 \equiv \mathbf{length}\ (pat_1\ ...\ pat_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\ \langle ne_1\ ne_2\ order\ t\rangle \vdash [pat_1, \ ..., pat_n] : \mathbf{vector}\ \langle ne_3\ ne_4\ order\ u\rangle \triangleright [pat_1', \ ..., pat_n'], (E^{\mathrm{T}}{}_1 \uplus\ ... \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime} \uplus \{ne_2 = ne_4\}}\ \text{CHECK\_PAT\_VECTOR}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad ... \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat_n', E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_1 \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_1 \quad ... \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_n \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_n \\ ne_4 \equiv \mathbf{length}\ (pat_1\ ...\ pat_n) \\ \mathbf{disjoint\ doms}\ (E^{\mathrm{T}}{}_1, \ ..., E^{\mathrm{T}}{}_n) \\ num_1 < ... < num_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\ \langle ne_1\ ne_2\ \mathbf{inc}\ t\rangle \vdash [num_1 = pat_1, \ ..., num_n = pat_n] : \mathbf{vector}\ \langle ne_3\ ne_4\ \mathbf{inc}\ t\rangle \triangleright [num_1 = pat_1', \ ..., num_n = pat_n'], (E^{\mathrm{T}}{}_1 \uplus\ ... \uplus E^{\mathrm{T}}{}_n), \{ne_1 \leq num_1, ne_2 \geq ne_4\} \uplus \Sigma$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad ... \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat_n', E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_1 \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_1 \quad ... \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_n \gtrapprox t : t', \Sigma^{\mathrm{N}\prime}{}_n \\ ne_4 \equiv \mathbf{length}\ (pat_1\ ...\ pat_n) \\ \mathbf{disjoint\ doms}\ (E^{\mathrm{T}}{}_1, \ ..., E^{\mathrm{T}}{}_n) \\ num_1 > ... > num_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus\ ... \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\ \langle ne_1\ ne_2\ \mathbf{dec}\ t\rangle \vdash [num_1 = pat_1, \ ..., num_n = pat_n] : \mathbf{vector}\ \langle ne_3\ ne_4\ \mathbf{dec}\ t\rangle \triangleright [num_1 = pat_1', \ ..., num_n = pat_n'], (E^{\mathrm{T}}{}_1 \uplus\ ... \uplus E^{\mathrm{T}}{}_n), \{ne_1 \geq num_1, ne_2 \geq ne_4\} \uplus$$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_1'' \; ne_1''' \; order \; t\rangle \vdash pat_1 : \mathbf{vector}\,\langle ne_1'' \; ne_1' \; order \; u_1\rangle \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad ... \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_n'' \; ne_n''' \; order \; t\rangle \vdash pat_1 : \mathbf{vector}\,\langle ne_n'' \; ne_n' \; order \; u_1\rangle \triangleright pat_n', E$
$E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_1 \gtrapprox t : t', \Sigma^{\mathrm{N}'}{}_1 \quad ... \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_n \gtrapprox t : t', \Sigma^{\mathrm{N}'}{}_n$
$\mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n)$
$\Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n$
$\Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}'}{}_n$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_1 \; ne_2 \; order \; t\rangle \vdash pat_1 : \ldots : pat_n : \mathbf{vector}\,\langle ne_1 \; ne_4 \; order \; t\rangle \triangleright pat_1' : \ldots : pat_n', (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{ne_1' + \ldots + ne_n' \leq ne_2\} \uplus \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}$

$$\frac{\begin{array}{l} E, t_1 \vdash pat_1 : u_1 \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .... \quad E, t_n \vdash pat_n : u_n \triangleright pat_n', E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \end{array}}{E, (t_1, \ldots, t_n) \vdash (pat_1, \ldots, pat_n) : (u_1, \ldots, u_n) \triangleright (pat_1', \ldots, pat_n'), (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n} \text{ CHECK\_PAT\_TUP}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat_1', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat_n', E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \gtrapprox t : t', \Sigma^{\mathrm{N}'}{}_1 \quad .. \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \gtrapprox t : t', \Sigma^{\mathrm{N}'}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle \vdash [||pat_1, .., pat_n||] : \mathbf{list}\,\langle t\rangle \triangleright [||pat_1', .., pat_n'||], (E^{\mathrm{T}}{}_1 \uplus .. \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}} \text{ CHECK\_PAT\_LIST}$$

$\boxed{E, t, widening \vdash exp : t' \triangleright exp', I, E^{\mathrm{T}}}$ Typing expressions, collecting nexp constraints, effects, and new bindings

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \{\}, \mathbf{Ctor}, \mathbf{unit} \to x\langle t\_args\rangle \; \mathbf{pure} \\ u \equiv x\langle t\_args\rangle[t\_arg_0/tid_0 \; .. \; t\_arg_n/tid_n] \\ E^{\mathrm{D}}, widening \vdash u \gtrapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : x \triangleright id, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, \{\}} \text{ CHECK\_EXP\_UNARYCTOR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\}, \{\}, tag, u \\ E^{\mathrm{D}}, widening, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}}, effect\rangle, \{\}} \text{ CHECK\_EXP\_LOCALVAR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u' \\ u \equiv u'[t\_arg_1/tid_1 \; .. \; t\_arg_n/tid_n] \\ E^{\mathrm{D}}, widening, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}'}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}, effect\rangle, \{\}} \text{ CHECK\_EXP\_OTHERVAR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \{\}, \mathbf{Ctor}, t'' \to x\langle t\_args\rangle\ \mathbf{pure} \\ t' \to u\ \mathbf{pure} \equiv t'' \to x\langle t\_args\rangle\ \mathbf{pure}[t\_arg_0/tid_0\ ..\ t\_arg_n/tid_n] \\ E^{\mathrm{D}}, widening \vdash u \precapprox t : t', \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash exp : u' \triangleright exp, \langle \Sigma^{\mathrm{N}\prime}, effect\rangle, E^{\mathrm{T}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id(exp) : t \triangleright id(exp'), \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}, effect\rangle, \{\}}\ \textsc{check\_exp\_ctor}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0\ ..\ t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ u_i \equiv (\mathbf{implicit}\ \langle ne\rangle, t_0, .., t_m) \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_0, .., t_m), widening \vdash (exp_0, .., exp_m) : u_i' \triangleright (exp_0', .., exp_m'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, widening, t \vdash id(annot, exp_0', .., exp_m') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id(exp_0, .., exp_m) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_appImplicit}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0\ ..\ t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, widening \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, widening, t \vdash id(exp') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id(exp) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\,\{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1\ ...\ tinf_n \\ u[t\_arg_0/tid_0\ ..\ t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, widening \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ \sigma_{\mathbf{full}(u_i', t)}(tinf_1\ ...\ tinf_n) \triangleright tinf \\ \langle (\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t, widening \vdash id(exp) : t' \triangleright exp'', I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id(exp) : u_j \triangleright exp'', I \uplus I' \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0\ ..\ t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, widening \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, widening, t \vdash exp_1'\ id\ exp_2' : u_j \triangleright u_j', exp, \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_infix\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\,\{tid_0 \mapsto kinf_0,\, ..,\, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \,...\, tinf_n \\ u[t\_arg_0/tid_0\, ..\, t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, widening \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ \sigma_{\mathbf{full}(u_i',t)}(tinf_1 \,...\, tinf_n) \triangleright tinf \\ \langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t, widening \vdash exp_1\ id\ exp_2 : t' \triangleright exp, I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_infix\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \triangleright x\langle t\_args\rangle, \overline{t_i}^{\,i} \\ \dfrac{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i, widening \vdash exp_i : u_i \triangleright exp_i', \langle \Sigma^{\mathrm{N}}{}_i, effect_i\rangle, E^{\mathrm{T}}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u_i \precapprox t_i : t_i', \Sigma^{\mathrm{N}\prime}{}_i}{}^{\,i} \\ \Sigma^{\mathrm{N}} \equiv \uplus\,\overline{\Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \Sigma^{\mathrm{N}\prime} \equiv \uplus\,\overline{\Sigma^{\mathrm{N}\prime}{}_i}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash \{\overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t\_args\rangle \triangleright \{\overline{id_i = exp_i'}^{\,i}\,;^?\},\, \uplus\langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime},\, \uplus\,\overline{effect_i}^{\,i}\rangle, \{\,\}}\ \textsc{check\_exp\_record}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash exp : x\langle t\_args\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E^{\mathrm{R}}(x\langle t\_args\rangle) \triangleright \overline{id_n' : t_n'}^{\,n} \\ \dfrac{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i, widening \vdash exp_i : u_i \triangleright exp_i', I_i, E^{\mathrm{T}}}{\overline{id_i : t_i}^{\,i} \subset \overline{id_n' : t_n'}^{\,n}}{}^{\,i} \\ \dfrac{}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u_i \precapprox t_i : t_i'', \Sigma^{\mathrm{N}\prime}{}_i}{}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash \{exp\ \mathbf{with}\ \overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t\_args\rangle \triangleright \{exp'\ \mathbf{with}\ \overline{id_i = exp_i'}^{\,i}\,\}, I \uplus \overline{I_i}^{\,i}, E^{\mathrm{T}}}\ \textsc{check\_exp\_recup}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \triangleright exp_1', I_1, E^{\mathrm{T}\prime} \quad ... \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash exp_n : u_n \triangleright exp_n', I_n, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \precapprox t : t', \Sigma^{\mathrm{N}}{}_1 \quad ... \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \precapprox t : t', \Sigma^{\mathrm{N}}{}_n \\ \mathbf{length}\,(exp_1 \,...\, exp_n) \equiv ne \\ \Sigma^{\mathrm{N}} \equiv \{ne = ne_2\} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \,...\, \uplus \Sigma^{\mathrm{N}}{}_n \end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp_1, \,...\,, exp_n] : \mathbf{vector}\,\langle ne_1\ num\ order\ t\rangle \triangleright [exp_1', \,...\,, exp_n'], \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle \uplus I_1 \uplus \,...\, \uplus I_n, E^{\mathrm{T}}}\ \textsc{check\_exp\_vector}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\ ne'\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_1\ ne_1'\ \mathbf{inc}\ u\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_2\ ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_3\ ne_3'\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \end{array}}{E, t, widening \vdash exp_1[exp_2] : u \triangleright exp_1'[exp_2'], I_1 \uplus I_2 \uplus \langle\{ne_1 \le ne_3, ne_3 + ne_3' \le ne_1 + ne_1'\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_vectorgetInc}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne\ ne'\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_1\ ne_1'\,\mathbf{dec}\ u\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_2\ ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_3\ ne_3'\rangle \triangleright exp_2', I_2, E^{\mathrm{T}}\end{array}}{E, t, widening \vdash exp_1[exp_2] : u \triangleright exp_1'[exp_2'], I_1 \uplus I_2 \uplus \langle\{ne_1 \geq ne_3, ne_3 + (-ne_3') \leq ne_1 + (-ne_1')\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp\_vectorgetDec}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne_1\ ne_1'\,\mathbf{inc}\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_2\ ne_2'\,\mathbf{inc}\ u\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\ ne_3'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_4\ ne_4'\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\ ne_5'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_3 : \mathbf{range}\,\langle ne_6\ ne_6'\rangle \triangleright exp_3', I_3, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{inc}\ t\rangle, widening \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\ ne_7'\,\mathbf{inc}\ u\rangle \triangleright exp_1'[exp_2' : exp_3'], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \geq ne_4, ne \leq ne_4', ne' \leq ne_4 + ne_6', ne_4 \leq ne_2, ne_4 + ne_6' \leq}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne_1\ ne_1'\,\mathbf{dec}\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_2\ ne_2'\,\mathbf{dec}\ u\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\ ne_3'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_4\ ne_4'\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\ ne_5'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_3 : \mathbf{range}\,\langle ne_6\ ne_6'\rangle \triangleright exp_3', I_3, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{dec}\ t\rangle, widening \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\ ne_7'\,\mathbf{dec}\ u\rangle \triangleright exp_1'[exp_2' : exp_3'], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \leq ne_4, ne \geq ne_4', ne' \leq ne_6' + (-ne_4), ne_4' \geq ne_2, ne_6' + (-}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{inc}\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_1\ ne_2\,\mathbf{inc}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_1'\ ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{range}\,\langle ne_3\ ne_4\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, t, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_2 : u \triangleright exp_2', I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{inc}\ t\rangle, widening \vdash [exp\,\mathbf{with}\ exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\ ne_2\,\mathbf{inc}\ u\rangle \triangleright [exp'\,\mathbf{with}\ exp_1' = exp_2'], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \leq ne_3, ne_2 \geq ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\ \textsc{check\_exp}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{dec}\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_1\ ne_2\,\mathbf{dec}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_1'\ ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{range}\,\langle ne_3\ ne_4\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, t, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_2 : u \triangleright exp_2', I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle ne\ ne'\,\mathbf{dec}\ t\rangle, widening \vdash [exp\,\mathbf{with}\ exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\ ne_2\,\mathbf{dec}\ u\rangle \triangleright [exp'\,\mathbf{with}\ exp_1' = exp_2'], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \geq ne_3, ne_2 \geq ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\ \textsc{check\_exi}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\,\mathbf{inc}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{vector}\,\langle ne_9\ ne_{10}\,\mathbf{inc}\ t\rangle, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : \mathbf{vector}\,\langle ne_{11}\ ne_{12}\,\mathbf{inc}\ u\rangle \triangleright exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_3 \leq ne_5, ne_3 + ne_4 \leq ne_7, ne_{12} = ne_8 + (-ne_6), ne_6 + \mathbf{one} \leq ne_8\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\,\mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\,\mathbf{inc}\ u\rangle \triangleright [exp'\,\mathbf{with}\ exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \textsc{check\_exp\_vecran}$$

$$\frac{\begin{array}{r}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\,\mathbf{inc}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, u, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : u' \triangleright exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_3 \leq ne_5, ne_3 + ne_4 \leq ne_7\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\,\mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\,\mathbf{inc}\ u\rangle \triangleright [exp'\,\mathbf{with}\ exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \textsc{check\_exp\_vecran}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{vector}\,\langle ne_9\ ne_{10}\ \mathbf{dec}\ t\rangle, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : \mathbf{vector}\,\langle ne_{11}\ ne_{12}\ \mathbf{dec}\ u\rangle \triangleright exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_5 \leq ne_3, ne_3 + (-ne_4) \leq ne_6 + (-ne_8), ne_8 + \mathbf{one} \leq ne_6\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\ \mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \triangleright [exp'\ \mathbf{with}\ exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_VECRAN}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \triangleright exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \triangleright exp_2', I_2, E^{\mathrm{T}} \\ E, u, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : u' \triangleright exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_5 \leq ne_3, ne_3 + (-ne_4) \leq ne_6 + (-ne_8), ne_8 + \mathbf{one} \leq ne_6\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\ \mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \triangleright [exp'\ \mathbf{with}\ exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_VECRAN}$$

$$\frac{\begin{array}{l}E^{\mathrm{R}}(x\langle t\_args\rangle) \triangleright \overline{id_i : t_i}^{\,i}\ id : u\ \overline{id_j' : t_j'}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t'', widening \vdash exp : x\langle t\_args\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E^{\mathrm{D}}, widening, t \vdash exp'.id : u \triangleright t', exp_1', \Sigma^{\mathrm{N}\prime}, effect\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash exp.id : u \triangleright exp_1', I \uplus \langle \Sigma^{\mathrm{N}\prime}, effect\rangle, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_FIELD}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'', widening \vdash exp : u \triangleright exp', I, E^{\mathrm{T}} \\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash pat_i : u_i' \triangleright pat_i', {E^{\mathrm{T}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_i), E^{\mathrm{D}}\rangle, t, widening \vdash exp_i : u_i'' \triangleright exp_i', I_i, {E^{\mathrm{T}}}_i'}^{\,i}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash \mathbf{switch}\ exp\{\ \overline{\mathbf{case}\ pat_i \to exp_i}^{\,i}\ \} : u \triangleright \mathbf{switch}\ exp'\{\ \overline{\mathbf{case}\ pat_i' \to exp_i'}^{\,i}\ \}, I \uplus \overline{I_i \uplus \langle {\Sigma^{\mathrm{N}}}_i, \mathbf{pure}\rangle}^{\,i}, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_CASE}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'', widening \vdash exp : u \triangleright exp', I, E^{\mathrm{T}} \\ E^{\mathrm{D}} \vdash typ \rightsquigarrow t' \\ E^{\mathrm{D}}, widening, t' \vdash exp' : u \triangleright u', exp'', \Sigma^{\mathrm{N}}, effect \\ E^{\mathrm{D}}, widening, t \vdash exp'' : t' \triangleright u'', exp''', \Sigma^{\mathrm{N}\prime}, effect'\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash (typ)exp : t \triangleright exp''', I \uplus \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, effect \uplus effect'\rangle, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_TYPED}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash letbind \triangleright letbind', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}}, effect, \{\,\} \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, t, widening \vdash exp : u \triangleright exp', I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash letbind\ \mathbf{in}\ exp : t \triangleright letbind'\ \mathbf{in}\ exp', \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus I_2, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_LET}$$

$$\frac{E, t_1, \mathit{widening} \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad .... \quad E, t_n, \mathit{widening} \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n}{E, (t_1, \ldots, t_n), \mathit{widening} \vdash (\mathit{exp}_1, \ldots, \mathit{exp}_n) : (u_1, \ldots, u_n) \triangleright (\mathit{exp}'_1, \ldots, \mathit{exp}'_n), I_1 \uplus .... \uplus I_n, E^{\mathrm{T}}} \text{ CHECK\_EXP\_TUP}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \lessapprox t : t', {\Sigma^{\mathrm{N}}}_1 \quad .. \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \lessapprox t : t', {\Sigma^{\mathrm{N}}}_n\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle, \mathit{widening} \vdash [||\mathit{exp}_1, .., \mathit{exp}_n||] : \mathbf{list}\,\langle u\rangle \triangleright [||\mathit{exp}'_1, .., \mathit{exp}'_n||], \langle {\Sigma^{\mathrm{N}}}_1 \uplus .. \uplus {\Sigma^{\mathrm{N}}}_n, \mathbf{pure}\rangle \uplus I_1 \uplus .. \uplus I_n, E^{\mathrm{T}}} \text{ CHECK\_EXP\_LIST}$$

$$\frac{\begin{array}{c}E, \mathbf{bit}, \mathit{widening} \vdash \mathit{exp}_1 : \mathbf{bit} \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}' \\ E, t, \mathit{widening} \vdash \mathit{exp}_2 : u_1 \triangleright \mathit{exp}'_2, I_2, {E^{\mathrm{T}}}_2 \\ E, t, \mathit{widening} \vdash \mathit{exp}_3 : u_2 \triangleright \mathit{exp}'_3, I_3, {E^{\mathrm{T}}}_3 \\ E^{\mathrm{D}}, \mathit{widening} \vdash u_1 \lessapprox t : t', {\Sigma^{\mathrm{N}}}_1 \\ E^{\mathrm{D}}, \mathit{widening} \vdash u_2 \lessapprox t : t', {\Sigma^{\mathrm{N}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathbf{if}\ \mathit{exp}_1\ \mathbf{then}\ \mathit{exp}_2\ \mathbf{else}\ \mathit{exp}_3 : u \triangleright \mathbf{if}\ \mathit{exp}'_1\ \mathbf{then}\ \mathit{exp}'_2\ \mathbf{else}\ \mathit{exp}'_3, \langle {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2, \mathbf{pure}\rangle \uplus I_1 \uplus I_2 \uplus I_3, ({E^{\mathrm{T}}}_2 \cap {E^{\mathrm{T}}}_3)} \text{ CHECK\_EXP\_IF}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2\rangle, \mathit{widening} \vdash \mathit{exp}_1 : \mathbf{range}\,\langle \mathit{ne}_7\ \mathit{ne}_8\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_3\ \mathit{ne}_4\rangle, \mathit{widening} \vdash \mathit{exp}_2 : \mathbf{range}\,\langle \mathit{ne}_9\ \mathit{ne}_{10}\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_5\ \mathit{ne}_6\rangle, \mathit{widening} \vdash \mathit{exp}_3 : \mathbf{range}\,\langle \mathit{ne}_{11}\ \mathit{ne}_{12}\rangle \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}} \\ \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_4\rangle\}), E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp}_4 : t \triangleright \mathit{exp}'_4, I_4, {E^{\mathrm{T}}}'\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}_1\ \mathbf{to}\ \mathit{exp}_2\ \mathbf{by}\ \mathit{exp}_3)\mathit{exp}_4 : t \triangleright \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}'_1\ \mathbf{to}\ \mathit{exp}'_2\ \mathbf{by}\ \mathit{exp}'_3)\mathit{exp}'_4, I_1 \uplus I_2 \uplus I_3 \uplus I_4 \uplus \langle \{\mathit{ne}_1 \leq \mathit{ne}_3 + \mathit{ne}_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \text{ C}$$

$$\frac{\begin{array}{c}E, t, \mathit{widening} \vdash \mathit{exp}_1 : u \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{list}\,\langle t\rangle, \mathit{widening} \vdash \mathit{exp}_2 : \mathbf{list}\,\langle u\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{list}\,\langle t\rangle, \mathit{widening} \vdash \mathit{exp}_1 :: \mathit{exp}_2 : \mathbf{list}\,\langle u\rangle \triangleright \mathit{exp}'_1 :: \mathit{exp}'_2, I_1 \uplus I_2, E^{\mathrm{T}}} \text{ CHECK\_EXP\_CONS}$$

$$\frac{\mathit{widening}, t \vdash \mathit{lit} : u \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}{E, t, \mathit{widening} \vdash \mathit{lit} : u \triangleright \mathit{exp}, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \text{ CHECK\_EXP\_LIT}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \triangleright \mathit{exp}', I, {E^{\mathrm{T}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \{\mathit{exp}\} : \mathbf{unit} \triangleright \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \text{ CHECK\_EXP\_BLOCKBASE}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \{\ \overline{\mathit{exp}_i}^{\,i}\ \} : \mathbf{unit} \triangleright \{\ \overline{\mathit{exp}'_i}^{\,i}\ \}, I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \{\mathit{exp};\ \overline{\mathit{exp}_i}^{\,i}\ \} : \mathbf{unit} \triangleright \{\mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i}\ \}, I_1 \uplus I_2, E^{\mathrm{T}}} \text{ CHECK\_EXP\_BLOCKREC}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \triangleright \mathit{exp}', I, {E^{\mathrm{T}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\,\{\mathit{exp}\} : \mathbf{unit} \triangleright \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \text{ CHECK\_EXP\_NONDETBASE}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\,\{\,\overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \triangleright \{\,\overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\,\{\mathit{exp};\, \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \triangleright \{\mathit{exp}';\, \overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \textsc{check\_exp\_nondetrec}$$

$$\frac{\begin{array}{c}E, t, \mathit{widening} \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ E, \mathit{widening} \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I_2, {E^{\mathrm{T}}}_2\end{array}}{E, \mathbf{unit}, \mathit{widening} \vdash \mathit{lexp} := \mathit{exp} : \mathbf{unit} \triangleright \mathit{lexp}' := \mathit{exp}', I \uplus I_2, {E^{\mathrm{T}}}_2} \quad \textsc{check\_exp\_assign}$$

$\boxed{E, \mathit{widening} \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I, E^{\mathrm{T}}}$ Check the left hand side of an assignment

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, \langle\{\,\}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wlocl}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_var}$$

$$\frac{\mathit{id} \notin \mathbf{dom}\,(E^{\mathrm{T}})}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, \{\mathit{id} \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnew}$$

$$\frac{\begin{array}{c}E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \lessapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}\textsc{Cast}$$

$$\frac{\begin{array}{c}E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \lessapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wlocl}\textsc{Cast}$$

$$\frac{\begin{array}{c}E^{\mathrm{T}}(\mathit{id}) \triangleright t \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \lessapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_var}\textsc{Cast}$$

$$\frac{\begin{array}{c} id \notin \mathbf{dom}\,(E^{\mathrm{T}}) \\ E^{\mathrm{D}} \vdash typ \rightsquigarrow t \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, widening \vdash (typ)id : t \triangleright id, I_\epsilon, \{id \mapsto \mathbf{reg}\,\langle t \rangle\}} \quad \textsc{check\_lexp\_wnewCast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, (t_1, \ldots, t_n, t) \rightarrow t' \,\{\, \overline{base\_effect_i}^{\,i}\,, \mathbf{wmem},\, \overline{base\_effect'_j}^{\,j}\,\} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, (t_1, \ldots, t_n), widening \vdash exp : u_1 \triangleright exp', I, {E^{\mathrm{T}}}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, widening \vdash id(exp) : t \triangleright id(exp'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wmem}\} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wmem}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, (t_1, \ldots, t_n, t) \rightarrow t' \,\{\, \overline{base\_effect_i}^{\,i}\,, \mathbf{wreg},\, \overline{base\_effect'_j}^{\,j}\,\} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, (t_1, \ldots, t_n), widening \vdash exp : u_1 \triangleright exp', I, {E^{\mathrm{T}}}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, widening \vdash id(exp) : t \triangleright id(exp'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wreg}\} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wregCall}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; t \rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle \{ne_1 \leq ne, ne_1 + ne_2 \geq ne\}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitInc}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; t \rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle \{ne \leq ne_1, ne_1 + (-ne_2) \leq ne\}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitDec}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne_1 \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2 \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; t \rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2 + (-ne_1)\; \mathbf{inc}\; t \rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle \{ne_3 \leq ne_1, ne_3 + ne_4 \leq ne_2 + (-ne_1)\}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wslic}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne_1 \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2 \rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; t \rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2 + (-ne_1)\; \mathbf{inc}\; t \rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle \{ne_1 \leq ne_3, ne_3 + (-ne_4) \leq ne_1 + (-ne_2)\}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_ws}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(x\langle t\_args \rangle) \triangleright \overline{id_i : t_i}^{\,i}\; id : t\; \overline{id'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \rangle, widening \vdash lexp : x\langle t\_args \rangle \triangleright lexp', I, E^{\mathrm{T}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \rangle, widening \vdash lexp.id : t \triangleright lexp'.id, I, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wrecord}$$

$\boxed{E \vdash letbind \triangleright letbind', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}, effect, E^{\mathrm{K}}}$ Build the environment for a let binding, collecting index constraints

$$\frac{\begin{array}{l}\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_2, \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, (\mathbf{none}, \mathbf{none}) \vdash \mathit{exp} : u' \triangleright \mathit{exp}', \langle {\Sigma^{\mathrm{N}}}_2, \mathit{effect}\rangle, {E^{\mathrm{T}}}_2 \\ \texttt{<<no parses (char 49): <E\_k u+ E\_k2,E\_a,E\_r,E\_e>,(none,none) |- u' < u,*** S\_N3 >>}\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{let}\ \mathit{typschm}\ \mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\ \mathit{typschm}\ \mathit{pat}' = \mathit{exp}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2 \uplus {\Sigma^{\mathrm{N}}}_3, \mathit{effect}, {E^{\mathrm{K}}}_2}\ \text{CHECK\_LETBIND\_VAL\_ANNOT}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \\ \texttt{<<no parses (char 26): <(E\_t u+ E\_t1),E\_d>,u |- e***xp : u' gives exp',<S\_N2,effect>,E\_t2 >>}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{let}\ \mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\ \mathit{pat}' = \mathit{exp}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2, \mathit{effect}, \{\,\}}\ \text{CHECK\_LETBIND\_VAL\_NOANNOT}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{type\_def} \triangleright E}$ Check a type definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \mathit{typschm} \triangleright \langle\{\,\}, \langle\{\,\}, \{\mathit{id} \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t\}, \{\,\}, \{\,\}\rangle\rangle}\ \text{CHECK\_TD\_ABBREV}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash \mathit{typ}_1 \rightsquigarrow t_1 \quad .. \quad E^{\mathrm{D}} \vdash \mathit{typ}_n \rightsquigarrow t_n \\ E^{\mathrm{R}} \equiv \{\{\mathit{id}_1 : t_1, .., \mathit{id}_n : t_n\} \mapsto x\}\end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ struct}\ \{\mathit{typ}_1\ \mathit{id}_1; ..; \mathit{typ}_n\ \mathit{id}_n\ ;^{?}\} \triangleright \langle\{\,\}, \langle\{x \mapsto K\_Typ\}, \{\,\}, E^{\mathrm{R}}, \{\,\}\rangle\rangle}\ \text{CHECK\_TD\_UNQUANT\_RECORD}$$

$$\frac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \langle E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \rightsquigarrow t_1 \quad .. \quad \langle E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \rightsquigarrow t_n \\ \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\} \equiv \uplus\, \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ E^{\mathrm{R}}_1 \equiv \{\{\mathit{id}_1 : t_1, .., \mathit{id}_n : t_n\} \mapsto \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\}, \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{None}, x\langle x'_1 .. x'_m\rangle\} \\ {E^{\mathrm{K}}}'_1 \equiv \{x \mapsto K\_Lam(k_1 .. k_m \to K\_Typ)\}\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ struct\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}.\{\mathit{typ}_1\ \mathit{id}_1; ..; \mathit{typ}_n\ \mathit{id}_n\ ;^{?}\} \triangleright \langle\{\,\}, \langle {E^{\mathrm{K}}}', \{\,\}, E^{\mathrm{R}}_1, \{\,\}\rangle\rangle}\ \text{CHECK\_TD\_QUANT\_RECORD}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_1 \to x\ \mathbf{pure}, \ldots, \mathit{id}_n \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_n \to x\ \mathbf{pure}\} \\ {E^{\mathrm{K}}}_1 \equiv \{x \mapsto K\_Typ\} \\ \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \rightsquigarrow t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \rightsquigarrow t_n\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ union}\ \{\mathit{typ}_1\ \mathit{id}_1; \ldots; \mathit{typ}_n\ \mathit{id}_n\ ;^{?}\} \triangleright \langle E^{\mathrm{T}}, \langle {E^{\mathrm{K}}}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle}\ \text{CHECK\_TD\_UNQUANT\_UNION}$$

$$\frac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \{x'_1 \mapsto k_1, \ldots, x'_m \mapsto k_m\} \equiv \uplus\, \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ E^{\mathrm{K}\prime} \equiv \{x \mapsto K\_Lam(k_1 \ldots k_m \to K\_Typ)\} \uplus\, \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \rightsquigarrow t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \rightsquigarrow t_n \\ t \equiv x\langle x'_1 \ldots x'_m\rangle \\ E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto E^{\mathrm{K}\prime}, \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{Ctor}, t_1 \to t\ \mathbf{pure}, \ldots, \mathit{id}_n \mapsto E^{\mathrm{K}\prime}, \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{Ctor}, t_n \to t\ \mathbf{pure}\}\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \mathbf{const\ union\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}.\{\mathit{typ}_1\ \mathit{id}_1; \ldots; \mathit{typ}_n\ \mathit{id}_n\ ;^?\} \triangleright \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, \{\,\}, \{\,\}, \{\,\}\rangle\rangle}\ \text{CHECK\_TD\_QUANT\_UNION}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto x, \ldots, \mathit{id}_n \mapsto x\} \\ E^{\mathrm{E}} \equiv \{x \mapsto \{\mathit{num}_1 \mapsto \mathit{id}_1 \ldots \mathit{num}_n \mapsto \mathit{id}_n\}\}\end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{enumerate}\,\{\mathit{id}_1; \ldots; \mathit{id}_n\ ;^?\} \triangleright \langle E^{\mathrm{T}}, \langle\{\mathit{id} \mapsto K\_Typ\}, \{\,\}, \{\,\}, E^{\mathrm{E}}\rangle\rangle}\ \text{CHECK\_TD\_ENUMERATE}$$

$\boxed{E \vdash \mathit{fundef} \triangleright \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Check a function definition

$$\frac{\begin{array}{l}E^{\mathrm{T}}(\mathit{id}) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect} \\ \overline{E^{\mathrm{D}} \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \Sigma^{\mathrm{N}\prime\prime} \equiv \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i} \\ E^{\mathrm{K}\prime} \equiv \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ {E^{\mathrm{D}}}_1 \equiv \langle E^{\mathrm{K}\prime}, \{\,\}, \{\,\}, \{\,\}\rangle \uplus E^{\mathrm{D}} \\ {E^{\mathrm{D}}}_1 \vdash \mathit{typ} \rightsquigarrow u \\ \texttt{<<no parses (char 12): E\_d1 |- u <*** t, S\_N2 >>} \\ \overline{\langle E^{\mathrm{T}}, {E^{\mathrm{D}}}_1\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}}}_j}^{\,j} \\ \texttt{<<no parses (char 29): </<(E\_t u+ E\_tj),E\_d1>,u |- e***xpj : u' gives expj',<S\_N'''j,effect'j>,E\_t'j//j/> >>} \\ \Sigma^{\mathrm{N}\prime\prime\prime\prime} \equiv {\Sigma^{\mathrm{N}}}_2 \uplus\, \overline{{\Sigma^{\mathrm{N}}}_j''' \uplus {\Sigma^{\mathrm{N}}}_j''''}^{\,j} \\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j} \\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime})\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\ rec\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}.\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\ rec\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}.\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}\ \text{CHECK\_FD\_REC}$$

$E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ effect$
$E^{\mathrm{D}} \vdash typ \rightsquigarrow u$
<<no parses (char 11): E_d |- u <*** t, S_N2 >>
$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash pat_j : u_j \triangleright pat', {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}}}''_j}^{\,j}$
<<no parses (char 28): </<(E_t u+ E_tj),E_d>,u |- e***xpj : uj' gives expj',<S_N''j,effect'j>,E_t'j//j/> >>
$effect \equiv \uplus \overline{effect'_j}^{\,j}$
$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}{}_2 \uplus \Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}}{}''_j \uplus \Sigma^{\mathrm{N}}{}'''_j}^{\,j})$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\ rec}\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{\,j} \triangleright \mathbf{function\ rec}\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$ CHECK_FD_REC_FUNCTION2

$\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash quant\_item_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i}$
$\Sigma^{\mathrm{N}\prime} \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}$
$E^{\mathrm{K}\prime} \equiv E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i}$
$\langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ \rightsquigarrow t$
$\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash pat_j : u_j \triangleright pat'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}}}''_j}^{\,j}$
$E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{id \mapsto E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ effect\})$
<<no parses (char 44): </<(E_t' u+ E_tj),<E_k',E_a,E_r,E_e>>,t |- e***xpj : u'j gives expj', <S_N''j,effect'j>,E_t'j//j/> >>
$effect \equiv \uplus \overline{effect'_j}^{\,j}$
$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}}{}''_j \uplus \Sigma^{\mathrm{N}}{}'''_j}^{\,j})$

$\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ rec\ forall}\ \overline{quant\_item_i}^{\,i}\ .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{\,j} \triangleright \mathbf{function\ rec\ forall}\ \overline{quant\_item_i}^{\,i}\ .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}$ C

$E^{\mathrm{D}} \vdash typ \rightsquigarrow t$
$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash pat_j : u_j \triangleright pat'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime}}_j}^{\,j}$
$E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{id \mapsto \{\,\}, \{\,\}, \mathbf{Global}, t_1 \to t\ effect\})$
<<no parses (char 29): </<(E_t' u+ E_tj),E_d>,t |- e***xpj : uj' gives expj', <S_N'j,effect'j>,E_t'j//j/> >>
$effect \equiv \uplus \overline{effect'_j}^{\,j}$
$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}}{}''_j}^{\,j})$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\ rec}\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{\,j} \triangleright \mathbf{function\ rec}\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}$ CHECK_FD_REC_FUNCTION_NO_SPEC

$E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ effect$
$\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash quant\_item_i \rightsquigarrow E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}$
$\Sigma^{\mathrm{N}\prime\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}$
$E^{\mathrm{K}\prime\prime} \equiv \overline{E^{\mathrm{K}}{}_i}^{\,i}$
$\langle E^{\mathrm{K}\prime\prime} \uplus E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ \rightsquigarrow u$
<<no parses (char 35): <E_k'' u+ E_k, E_a,E_r,E_e> |- u <*** t, S_N2 >>
$\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash pat_j : u_j \triangleright pat'_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}}{}''_j}^{\,j}$
<<no parses (char 57): </<(E_t u- id u+ E_tj),<E_k u+ E_k'',E_a,E_r,E_e>>,t |- e***xpj : uj' gives expj', <S_N'''j,effect'j>,E_t'j//j/> >>
$\Sigma^{\mathrm{N}\prime\prime\prime\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j}$
$effect \equiv \uplus\, \overline{effect'_j}^{\,j}$
$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime})$

$\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ forall}\ \overline{quant\_item_i}^{\,i}\ .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{\,j} \triangleright \mathbf{function\ forall}\ \overline{quant\_item_i}^{\,i}\ .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$ C

$E^{\mathrm{T}}(id) \triangleright \{\,\}, \Sigma^{\mathrm{N}}{}_1, \mathbf{Global}, t_1 \to t\ effect$
$E^{\mathrm{D}} \vdash typ \rightsquigarrow u$
<<no parses (char 11): E_d |- u <*** t, S_N2 >>
$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash pat_j : u_j \triangleright pat_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime}{}_j}^{\,j}$
<<no parses (char 36): </<(E_t u- id u+ E_tj),E_d>, u |- e***xpj : uj' gives expj', <S_N''j,effect'j>,E_t'j//j/> >>
$effect \equiv \uplus\, \overline{effect'_j}^{\,j}$
$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2 \uplus\ \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j})$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function}\ \ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{\,j} \triangleright \mathbf{function}\ \ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$ CHECK_FD_FUNCTION2

$$\frac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i} \\ E^{\mathrm{K}\prime\prime} \equiv E^{\mathrm{K}} \uplus\, \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \rightsquigarrow t \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime\prime}}_j}^{\,j} \\ E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\}) \\ \texttt{<<no parses (char 44): </<(E\_t u+ E\_tj),<E\_k'',E\_a,E\_r,E\_e>>,t |- e***xpj : uj' gives expj', <S\_N''j,effect'j>,E\_t'j//j/> >>} \\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j} \\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus\, \overline{{\Sigma^{\mathrm{N}\prime}}_j \uplus {\Sigma^{\mathrm{N}\prime\prime}}_j}^{\,j})\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\ \text{CHECK\_}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t \\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime}}_j}^{\,j} \\ E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \Sigma^{\mathrm{N}}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\}) \\ \texttt{<<no parses (char 28): </<(E\_t u+ E\_tj),E\_d>,t |- e***xpj : uj' gives exp', <S\_N'j,effect'j>,E\_t'j//j/> >>} \\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j} \\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{{\Sigma^{\mathrm{N}\prime}}_j \uplus {\Sigma^{\mathrm{N}\prime\prime}}_j}^{\,j})\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\ \text{CHECK\_FD\_FUNCTION\_NO\_SPEC2}$$

$\boxed{E \vdash \mathit{val\_spec} \triangleright E^{\mathrm{T}}}$ Check a value specification

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val}\ \mathit{typschm}\ \mathit{id} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}, \mathbf{Global}, t\}}\ \text{CHECK\_SPEC\_VAL\_SPEC}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val\ extern}\ \mathit{typschm}\ \mathit{id} = \mathit{string} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, t\}}\ \text{CHECK\_SPEC\_EXTERN}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{default\_spec} \triangleright E^{\mathrm{T}}, {E^{\mathrm{K}}}_1}$ Check a default typing specification

$$\frac{E^{\mathrm{K}} \vdash \mathit{base\_kind} \rightsquigarrow k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{default}\ \mathit{base\_kind}\ 'x \triangleright \{\,\}, \{'x \mapsto k\ \mathbf{default}\}}\ \text{CHECK\_DEFAULT\_KIND}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_1, {\Sigma^{\mathrm{N}}}}{E^{\mathrm{D}} \vdash \mathbf{default}\ \mathit{typschm}\ \mathit{id} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, {\Sigma^{\mathrm{N}}}, \mathbf{Default}, t\}, \{\,\}} \quad \text{CHECK\_DEFAULT\_TYP}$$

$\boxed{E \vdash \mathit{def} \triangleright \mathit{def}', E'}$ Check a definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{type\_def} \triangleright E}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathit{type\_def} \triangleright \mathit{type\_def}, \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \uplus E} \quad \text{CHECK\_DEF\_TDEF}$$

$$\frac{E \vdash \mathit{fundef} \triangleright \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}{E \vdash \mathit{fundef} \triangleright \mathit{fundef}', E \uplus \langle E^{\mathrm{T}}, \epsilon \rangle} \quad \text{CHECK\_DEF\_FDEF}$$

$$\frac{\begin{array}{l} E \vdash \mathit{letbind} \triangleright \mathit{letbind}', \{\mathit{id}_1 \mapsto t_1, \ldots, \mathit{id}_n \mapsto t_n\}, \Sigma^{\mathrm{N}}, \mathbf{pure}, E^{\mathrm{K}} \\ {\Sigma^{\mathrm{N}}}_1 \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}) \end{array}}{E \vdash \mathit{letbind} \triangleright \mathit{letbind}', E \uplus \langle \{\mathit{id}_1 \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_1, \ldots, \mathit{id}_n \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_n\}, \epsilon \rangle} \quad \text{CHECK\_DEF\_VDEF}$$

$$\frac{E \vdash \mathit{val\_spec} \triangleright E^{\mathrm{T}}}{E \vdash \mathit{val\_spec} \triangleright \mathit{val\_spec}, E \uplus \langle E^{\mathrm{T}}, \epsilon \rangle} \quad \text{CHECK\_DEF\_VSPEC}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{default\_spec} \triangleright {E^{\mathrm{T}}}_1, {E^{\mathrm{K}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathit{default\_spec} \triangleright \mathit{default\_spec}, \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}} \uplus \langle {E^{\mathrm{K}}}_1, \{\,\}, \{\,\}, \{\,\} \rangle \rangle} \quad \text{CHECK\_DEF\_DEFAULT}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathbf{register}\ \mathit{typ}\ \mathit{id} \triangleright \mathbf{register}\ \mathit{typ}\ \mathit{id}, \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{register}\ \langle t \rangle\}), E^{\mathrm{D}} \rangle} \quad \text{CHECK\_DEF\_REGISTER}$$

$\boxed{E \vdash \mathit{defs} \triangleright \mathit{defs}', E'}$ Check definitions, potentially given default environment of built-in library

$$\frac{\begin{array}{l} E \vdash \mathit{def} \triangleright \mathit{def}', E_1 \\ E \uplus E_1 \vdash \overline{\mathit{def}_i}^{\,i} \triangleright \overline{\mathit{def}'_i}^{\,i}, E_2 \end{array}}{E \vdash \mathit{def}\ \overline{\mathit{def}_i}^{\,i} \triangleright \mathit{def}'\ \overline{\mathit{def}'_i}^{\,i}, E_2} \quad \text{CHECK\_DEFS\_DEFS}$$

4 Sail operational semantics {TODO}