```
Module No5.

Import No1.
Import No2.
Import No3.
Import No4.

Theorem n5_1 : ∀ P Q : Prop,
  (P ∧ Q) → (P ↔ Q).
  Proof. intros P Q.
  specialize n3_4 with P Q.
  intros n3_4a.
  specialize n3_4 with Q P.
  intros n3_4b.
  specialize n3_22 with P Q.
  intros n3_22a.
  Syll n3_22a n3_4b Sa.
  clear n3_22a. clear n3_4b.
  Conj n3_4a Sa.
  split.
  apply n3_4a.
  apply Sa.
  specialize n4_76 with (P∧Q) (P→Q) (Q→P).
  intros n4_76a.
  replace ((P ∧ Q → P → Q) ∧ (P ∧ Q → Q → P)) with (P ∧ Q → (P → Q) ∧ (Q
→ P)) in H.
  replace ((P→Q)∧(Q→P)) with (P↔Q) in H.
  apply H.
  apply Equiv4_01.
  replace (P ∧ Q → (P → Q) ∧ (Q → P)) with ((P ∧ Q → P → Q) ∧ (P ∧ Q → Q
→ P)).
  reflexivity.
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 apply EqBi.
 apply n4_76a.
 Qed. (*Note that n4_76 is not cited, but it is used to move from ((a→b) ∧ (a→c)) to (a→ (b ∧ c)).*)

Theorem n5_11 : ∀ P Q : Prop,
 (P → Q) ∨ (~P → Q).
 Proof. intros P Q.
 specialize n2_5 with P Q.
 intros n2_5a.
 specialize n2_54 with ((P → Q)) (~ P → Q).
 intros n2_54a.
 MP n2_54a n2_5a.
 apply n2_54a.
 Qed.

Theorem n5_12 : ∀ P Q : Prop,
 (P → Q) ∨ (P → ~Q).
 Proof. intros P Q.
 specialize n2_51 with P Q.
 intros n2_51a.
 specialize n2_54 with ((P → Q)) (P → ~ Q).
 intros n2_54a.
 MP n2_54a n2_5a.
 apply n2_54a.
 Qed.

Theorem n5_13 : ∀ P Q : Prop,
 (P → Q) ∨ (Q → P).
 Proof. intros P Q.
 specialize n2_521 with P Q.
 intros n2_521a.
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 replace (~ (P → Q) → Q → P) with (~~ (P → Q) ∨ (Q → P)) in n2_521a.
 replace (~~(P→Q)) with (P→Q) in n2_521a.
 apply n2_521a.
 apply EqBi.
 apply n4_13.
 replace (~~ (P → Q) ∨ (Q → P)) with (~ (P → Q) → Q → P).
 reflexivity.
 apply Impl1_01.
 Qed. (*n4_13 is not cited, but is needed for double negation elimination.
*)

Theorem n5_14 : ∀ P Q R : Prop,
 (P → Q) ∨ (Q → R).
 Proof. intros P Q R.
 specialize n2_02 with P Q.
 intros n2_02a.
 specialize Trans2_16 with Q (P→Q).
 intros Trans2_16a.
 MP Trans2_16a n2_02a.
 specialize n2_21 with Q R.
 intros n2_21a.
 Syll Trans2_16a n2_21a Sa.
 replace (~(P→Q)→(Q→R)) with (~~(P→Q)∨(Q→R)) in Sa.
 replace (~~(P→Q)) with (P→Q) in Sa.
 apply Sa.
 apply EqBi.
 apply n4_13.
 replace (~~(P→Q)∨(Q→R)) with (~(P→Q)→(Q→R)).
 reflexivity.
 apply Impl1_01.
 Qed.
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Theorem n5_15 : ∀ P Q : Prop,
  (P ↔ Q) ∨ (P ↔ ~Q).
  Proof. intros P Q.
  specialize n4_61 with P Q.
  intros n4_61a.
  replace (~ (P → Q) ↔ P ∧ ~ Q) with ((~ (P → Q) → P ∧ ~ Q) ∧ ((P ∧ ~ Q)
→ ~ (P → Q))) in n4_61a.
  specialize Simp3_26 with (~ (P → Q) → P ∧ ~ Q) ((P ∧ ~ Q) → ~ (P → Q)
).
  intros Simp3_26a.
  MP Simp3_26a n4_61a.
  specialize n5_1 with P (~Q).
  intros n5_1a.
  Syll Simp3_26a n5_1a Sa.
  specialize n2_54 with (P→Q) (P ↔ ~Q).
  intros n2_54a.
  MP n2_54a Sa.
  specialize n4_61 with Q P.
  intros n4_61b.
  replace ((~(Q → P)) ↔ (Q ∧ ¬ P)) with (((~(Q → P)) → (Q ∧ ¬ P)) ∧ ((Q ∧
 ¬ P) → (~(Q → P)))) in n4_61b.
  specialize Simp3_26 with (~(Q → P)→ (Q ∧ ¬ P)) ((Q ∧ ¬ P)→ (~(Q → P))
).
  intros Simp3_26b.
  MP Simp3_26b n4_61b.
  specialize n5_1 with Q (~P).
  intros n5_1b.
  Syll Simp3_26b n5_1b Sb.
  specialize n4_12 with P Q.
  intros n4_12a.
  replace (Q↔~P) with (P↔~Q) in Sb.
  specialize n2_54 with (Q→P) (P↔~Q).
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intros n2_54b.
MP n2_54b Sb.
clear n4_61a. clear Simp3_26a. clear n5_1a. clear n2_54a. clear n4_61b. c
lear Simp3_26b. clear n5_1b. clear n4_12a. clear n2_54b.
replace (¬ (P → Q) → P ↔ ¬ Q) with (~~ (P → Q) ∨ (P ↔ ¬ Q)) in Sa.
replace (~~(P→Q)) with (P→Q) in Sa.
replace (¬ (Q → P) → (P ↔ ~Q)) with (~~(Q → P) ∨ (P ↔ ~Q)) in Sb.
replace (~~(Q→P)) with (Q→P) in Sb.
Conj Sa Sb.
split.
apply Sa.
apply Sb.
specialize n4_41 with (P↔~Q) (P→Q) (Q→P).
intros n4_41a.
replace ((P → Q) ∨ (P ↔ ¬ Q)) with ((P ↔ ¬ Q) ∨ (P → Q)) in H.
replace ((Q → P) ∨ (P ↔ ¬ Q)) with ((P ↔ ¬ Q) ∨ (Q → P)) in H.
replace (((P ↔ ¬ Q) ∨ (P → Q)) ∧ ((P ↔ ¬ Q) ∨ (Q → P))) with ((P ↔ ¬ Q)
∨ (P → Q) ∧ (Q → P)) in H.
replace ((P→Q) ∧ (Q → P)) with (P↔Q) in H.
replace ((P ↔ ¬ Q) ∨ (P ↔ Q)) with ((P ↔ Q) ∨ (P ↔ ¬ Q)) in H.
apply H.
apply EqBi.
apply n4_31.
apply Equiv4_01.
apply EqBi.
apply n4_41a.
apply EqBi.
apply n4_31.
apply EqBi.
apply n4_31.
apply EqBi.
apply n4_13.
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 replace (~~(Q → P) ∨ (P ↔ ~Q)) with (¬ (Q → P) → (P ↔ ~Q)).
 reflexivity.
 apply Impl1_01.
 apply EqBi.
 apply n4_13.
 replace (~~ (P → Q) ∨ (P ↔ ¬ Q)) with (¬ (P → Q) → P ↔ ¬ Q).
 reflexivity.
 apply Impl1_01.
 apply EqBi.
 apply n4_12a.
 apply Equiv4_01.
 apply Equiv4_01.
 Qed.

Theorem n5_16 : ∀ P Q : Prop,
 ~((P ↔ Q) ∧ (P ↔ ~Q)).
 Proof. intros P Q.
 specialize Simp3_26 with ((P→Q)∧ (P → ¬ Q)) (Q→P).
 intros Simp3_26a.
 specialize n2_08 with ((P ↔ Q) ∧ (P → ~Q)).
 intros n2_08a.
 replace (((P → Q) ∧ (P → ¬ Q)) ∧ (Q → P)) with ((P → Q) ∧ ((P → ¬ Q) ∧ (
Q → P))) in Simp3_26a.
 replace ((P → ¬ Q) ∧ (Q → P)) with ((Q → P) ∧ (P → ¬ Q)) in Simp3_26a.
 replace ((P→Q) ∧ (Q → P)∧ (P → ~Q)) with (((P→Q) ∧ (Q → P)) ∧ (P → ~
Q)) in Simp3_26a.
 replace ((P → Q) ∧ (Q → P)) with (P↔Q) in Simp3_26a.
 Syll n2_08a Simp3_26a Sa.
 specialize n4_82 with P Q.
 intros n4_82a.
 replace ((P → Q) ∧ (P → ¬ Q)) with (~P) in Sa.
 specialize Simp3_27 with (P→Q) ((Q→P)∧ (P → ¬ Q)).
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 intros Simp3_27a.
 replace ((P→Q) ∧ (Q → P)∧ (P → ~Q)) with (((P→Q) ∧ (Q → P)) ∧ (P → ~
Q)) in Simp3_27a.
 replace ((P → Q) ∧ (Q → P)) with (P↔Q) in Simp3_27a.
 specialize Syll3_33 with Q P (~Q).
 intros Syll3_33a.
 Syll Simp3_27a Syll2_06a Sb.
 specialize Abs2_01 with Q.
 intros Abs2_01a.
 Syll Sb Abs2_01a Sc.
 clear Sb. clear Simp3_26a. clear n2_08a. clear n4_82a. clear Simp3_27a. c
lear Syll3_33a. clear Abs2_01a.
 Conj Sa Sc.
 split.
 apply Sa.
 apply Sc.
 specialize Comp3_43 with ((P ↔ Q) ∧ (P → ¬ Q)) (~P) (~Q).
 intros Comp3_43a.
 MP Comp3_43a H.
 specialize n4_65 with Q P.
 intros n4_65a.
 replace (¬ Q ∧ ¬ P) with (¬ P ∧ ¬ Q) in n4_65a.
 replace (¬ P ∧ ¬ Q) with (~(~Q→P)) in Comp3_43a.
 specialize Exp3_3 with (P↔Q) (P→~Q) (~(~Q→P)).
 intros Exp3_3a.
 MP Exp3_3a Comp3_43a.
 replace ((P→~Q)→~(~Q→P)) with (~(P→~Q)∨~(~Q→P)) in Exp3_3a.
 specialize n4_51 with (P→~Q) (~Q→P).
 intros n4_51a.
 replace (¬ (P → ¬ Q) ∨ ¬ (¬ Q → P)) with (¬ ((P → ¬ Q) ∧ (¬ Q → P))) in E
xp3_3a.
 replace ((P→~Q) ∧ (~ Q → P)) with (P↔~Q) in Exp3_3a.
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replace ((P↔Q)→~(P↔~Q)) with (~(P↔Q)∨~(P↔~Q)) in Exp3_3a.
specialize n4_51 with (P↔Q) (P↔~Q).
intros n4_51b.
replace (¬ (P ↔ Q) ∨ ¬ (P ↔ ¬ Q)) with (¬ ((P ↔ Q) ∧ (P ↔ ¬ Q))) in Exp
3_3a.
apply Exp3_3a.
apply EqBi.
apply n4_51b.
replace (¬ (P ↔ Q) ∨ ¬ (P ↔ ¬ Q)) with (P ↔ Q → ¬ (P ↔ ¬ Q)).
reflexivity.
apply Impl1_01.
apply Equiv4_01.
apply EqBi.
apply n4_51a.
replace (¬ (P → ¬ Q) ∨ ¬ (¬ Q → P)) with ((P → ¬ Q) → ¬ (¬ Q → P)).
reflexivity.
apply Impl1_01.
apply EqBi.
apply n4_65a.
apply EqBi.
apply n4_3.
apply Equiv4_01.
apply EqBi.
apply n4_32.
replace (¬ P) with ((P → Q) ∧ (P → ¬ Q)).
reflexivity.
apply EqBi.
apply n4_82a.
apply Equiv4_01.
apply EqBi.
apply n4_32.
apply EqBi.
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 apply n4_3.
 replace ((P → Q) ∧ (P → ¬ Q) ∧ (Q → P)) with (((P → Q) ∧ (P → ¬ Q)) ∧ (Q
→ P)).
 reflexivity.
 apply EqBi.
 apply n4_32.
 Qed.

Theorem n5_17 : ∀ P Q : Prop,
 ((P ∨ Q) ∧ ~(P ∧ Q)) ↔ (P ↔ ~Q).
 Proof. intros P Q.
 specialize n4_64 with Q P.
 intros n4_64a.
 specialize n4_21 with (Q∨P) (~Q→P).
 intros n4_21a.
 replace ((~Q→P)↔(Q∨P)) with ((Q∨P)↔(~Q→P)) in n4_64a.
 replace (Q∨P) with (P∨Q) in n4_64a.
 specialize n4_63 with P Q.
 intros n4_63a.
 replace (¬ (P → ¬ Q) ↔ P ∧ Q) with (P ∧ Q ↔ ¬ (P → ¬ Q)) in n4_63a.
 specialize Trans4_11 with (P∧Q) (~(P→~Q)).
 intros Trans4_11a.
 replace (~~(P→~Q)) with (P→~Q) in Trans4_11a.
 replace (P ∧ Q ↔ ¬ (P → ¬ Q)) with (¬ (P ∧ Q) ↔ (P → ¬ Q)) in n4_63a.
 clear Trans4_11a. clear n4_21a.
 Conj n4_64a n4_63a.
 split.
 apply n4_64a.
 apply n4_63a.
 specialize n4_38 with (P ∨ Q) (¬ (P ∧ Q)) (¬ Q → P) (P → ¬ Q).
 intros n4_38a.
 MP n4_38a H.
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replace ((~Q→P) ∧ (P → ~Q)) with (~Q↔P) in n4_38a.
specialize n4_21 with P (~Q).
intros n4_21b.
replace (~Q↔P) with (P↔~Q) in n4_38a.
apply n4_38a.
apply EqBi.
apply n4_21b.
apply Equiv4_01.
replace (¬ (P ∧ Q) ↔ (P → ¬ Q)) with (P ∧ Q ↔ ¬ (P → ¬ Q)).
reflexivity.
apply EqBi.
apply Trans4_11a.
apply EqBi.
apply n4_13.
apply EqBi.
apply n4_21.
apply EqBi.
apply n4_31.
apply EqBi.
apply n4_21a.
Qed.

Theorem n5_18 : ∀ P Q : Prop,
(P ↔ Q) ↔ ~(P ↔ ~Q).
Proof. intros P Q.
specialize n5_15 with P Q.
intros n5_15a.
specialize n5_16 with P Q.
intros n5_16a.
Conj n5_15a n5_16a.
split.
apply n5_15a.
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 apply n5_16a.
 specialize n5_17 with (P↔Q) (P↔~Q).
 intros n5_17a.
 replace ((P ↔ Q) ↔ ¬ (P ↔ ¬ Q)) with (((P ↔ Q) ∨ (P ↔ ¬ Q)) ∧ ¬ ((P ↔
Q) ∧ (P ↔ ¬ Q))).
 apply H.
 apply EqBi.
 apply n5_17a.
 Qed.

Theorem n5_19 : ∀ P : Prop,
 ~(P ↔ ~P).
 Proof. intros P.
 specialize n5_18 with P P.
 intros n5_18a.
 specialize n4_2 with P.
 intros n4_2a.
 replace (~(P↔~P)) with (P↔P).
 apply n4_2a.
 apply EqBi.
 apply n5_18a.
 Qed.

Theorem n5_21 : ∀ P Q : Prop,
 (~P ∧ ~Q) → (P ↔ Q).
 Proof. intros P Q.
 specialize n5_1 with (~P) (~Q).
 intros n5_1a.
 specialize Trans4_11 with P Q.
 intros Trans4_11a.
 replace (~P↔~Q) with (P↔Q) in n5_1a.
 apply n5_1a.
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  apply EqBi.
  apply Trans4_11a.
  Qed.

Theorem n5_22 : ∀ P Q : Prop,
  ~(P ↔ Q) ↔ ((P ∧ ~Q) ∨ (Q ∧ ~P)).
  Proof. intros P Q.
  specialize n4_61 with P Q.
  intros n4_61a.
  specialize n4_61 with Q P.
  intros n4_61b.
  Conj n4_61a n4_61b.
  split.
  apply n4_61a.
  apply n4_61b.
  specialize n4_39 with (~(P → Q)) (~(Q → P)) (P ∧ ~Q) (Q ∧ ~P).
  intros n4_39a.
  MP n4_39a H.
  specialize n4_51 with (P→Q) (Q→P).
  intros n4_51a.
  replace (~(P → Q) ∨ ~(Q → P)) with (~((P → Q) ∧ (Q → P))) in n4_39a.
  replace ((P → Q) ∧ (Q → P)) with (P↔Q) in n4_39a.
  apply n4_39a.
  apply Equiv4_01.
  apply EqBi.
  apply n4_51a.
  Qed.

Theorem n5_23 : ∀ P Q : Prop,
  (P ↔ Q) ↔ ((P ∧ Q) ∨ (~P ∧ ~Q)).
  Proof. intros P Q.
  specialize n5_18 with P Q.
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 intros n5_18a.
 specialize n5_22 with P (~Q).
 intros n5_22a.
 specialize n4_13 with Q.
 intros n4_13a.
 replace (~(P↔~Q)) with ((P ∧ ~~Q) ∨ (~Q ∧ ~P)) in n5_18a.
 replace (~~Q) with Q in n5_18a.
 replace (~Q ∧ ~P) with (~P ∧ ~Q) in n5_18a.
 apply n5_18a.
 apply EqBi.
 apply n4_3. (*with (~P) (~Q)*)
 apply EqBi.
 apply n4_13a.
 replace (P∧~~Q∨~Q∧~P) with (~(P↔~Q)).
 reflexivity.
 apply EqBi.
 apply n5_22a.
 Qed. (*The proof sketch in Principia offers n4_36, but we found it far si
mpler to simply use the commutativity of conjunction (n4_3).*)

Theorem n5_24 : ∀ P Q : Prop,
 ~((P ∧ Q) ∨ (~P ∧ ~Q)) ↔ ((P ∧ ~Q) ∨ (Q ∧ ~P)).
 Proof. intros P Q.
 specialize n5_22 with P Q.
 intros n5_22a.
 specialize n5_23 with P Q.
 intros n5_23a.
 replace ((P↔Q)↔((P∧ Q) ∨(~P ∧ ~Q))) with ((~(P↔Q)↔~((P∧ Q) ∨(~
P ∧ ~Q)))) in n5_23a.
 replace (~(P↔Q)) with (~((P ∧ Q) ∨ (~P ∧ ~Q))) in n5_22a.
 apply n5_22a.
 replace (~((P ∧ Q) ∨ (~P ∧ ~Q))) with (~(P ↔ Q)).
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 reflexivity.
 apply EqBi.
 apply n5_23a.
 replace (~(P ↔ Q) ↔ ~(P ∧ Q ∨ ~P ∧ ~Q)) with ((P ↔ Q) ↔ P ∧ Q ∨ ~P
∧ ~Q).
 reflexivity.
 specialize Trans4_11 with (P↔Q) (P ∧ Q ∨ ~P ∧ ~Q).
 intros Trans4_11a.
 apply EqBi.
 apply Trans4_11a.
 Qed. (*Note that Trans4_11 is not cited explicitly.*)

Theorem n5_25 : ∀ P Q : Prop,
 (P ∨ Q) ↔ ((P → Q) → Q).
 Proof. intros P Q.
 specialize n2_62 with P Q.
 intros n2_62a.
 specialize n2_68 with P Q.
 intros n2_68a.
 Conj n2_62a n2_68a.
 split.
 apply n2_62a.
 apply n2_68a.
 Equiv H.
 apply H.
 apply Equiv4_01.
 Qed.

Theorem n5_3 : ∀ P Q R : Prop,
 ((P ∧ Q) → R) ↔ ((P ∧ Q) → (P ∧ R)).
 Proof. intros P Q R.
 specialize Comp3_43 with (P ∧ Q) P R.
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 intros Comp3_43a.
 specialize Exp3_3 with (P ∧ Q → P) (P ∧ Q →R) (P ∧ Q → P ∧ R).
 intros Exp3_3a.
 MP Exp3_3a Comp3_43a.
 specialize Simp3_26 with P Q.
 intros Simp3_26a.
 MP Exp3_3a Simp3_26a.
 specialize Syll2_05 with (P ∧ Q) (P ∧ R) R.
 intros Syll2_05a.
 specialize Simp3_27 with P R.
 intros Simp3_27a.
 MP Syll2_05a Simp3_27a.
 clear Comp3_43a. clear Simp3_27a. clear Simp3_26a.
 Conj Exp3_3a Syll2_05a.
 split.
 apply Exp3_3a.
 apply Syll2_05a.
 Equiv H.
 apply H.
 apply Equiv4_01.
 Qed. (*Note that Exp is not cited in the proof sketch, but seems necessar
y.*)

Theorem n5_31 : ∀ P Q R : Prop,
 (R ∧ (P → Q)) → (P → (Q ∧ R)).
 Proof. intros P Q R.
 specialize Comp3_43 with P Q R.
 intros Comp3_43a.
 specialize n2_02 with P R.
 intros n2_02a.
 replace ((P→Q) ∧ (P→R)) with ((P→R) ∧ (P→Q)) in Comp3_43a.
 specialize Exp3_3 with (P→R) (P→Q) (P→(Q ∧ R)).
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 intros Exp3_3a.
 MP Exp3_3a Comp3_43a.
 Syll n2_02a Exp3_3a Sa.
 specialize Imp3_31 with R (P→Q) (P→(Q ∧ R)).
 intros Imp3_31a.
 MP Imp3_31a Sa.
 apply Imp3_31a.
 apply EqBi.
 apply n4_3. (*with (P→R)∧(P→Q)).*)
 Qed. (*Note that Exp, Imp, and n4_3 are not cited in the proof sketch.*)

Theorem n5_32 : ∀ P Q R : Prop,
 (P → (Q ↔ R)) ↔ ((P ∧ Q) ↔ (P ∧ R)).
 Proof. intros P Q R.
 specialize n4_76 with P (Q→R) (R→Q).
 intros n4_76a.
 specialize Exp3_3 with P Q R.
 intros Exp3_3a.
 specialize Imp3_31 with P Q R.
 intros Imp3_31a.
 Conj Exp3_3a Imp3_31a.
 split.
 apply Exp3_3a.
 apply Imp3_31a.
 Equiv H.
 specialize Exp3_3 with P R Q.
 intros Exp3_3b.
 specialize Imp3_31 with P R Q.
 intros Imp3_31b.
 Conj Exp3_3b Imp3_31b.
 split.
 apply Exp3_3b.
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apply Imp3_31b.
Equiv H0.
specialize n5_3 with P Q R.
intros n5_3a.
specialize n5_3 with P R Q.
intros n5_3b.
replace (P→Q→R) with (P∧Q→R) in n4_76a.
replace (P∧Q→R) with (P∧Q→P∧R) in n4_76a.
replace (P→R→Q) with (P∧R→Q) in n4_76a.
replace (P∧R→Q) with (P∧R→P∧Q) in n4_76a.
replace ((P∧Q→P∧R)∧(P∧R→P∧Q)) with ((P∧Q)↔(P∧R)) in n4_76a.
replace ((P∧Q ↔ P∧R)↔(P→(Q→R)∧(R→Q))) with ((P→(Q→R)∧(R→Q))
↔(P∧Q ↔ P∧R)) in n4_76a.
replace ((Q→R)∧(R→Q)) with (Q↔R) in n4_76a.
apply n4_76a.
apply Equiv4_01.
apply EqBi.
apply n4_3. (*to commute the biconditional to get the theorem.*)
apply Equiv4_01.
replace (P ∧ R → P ∧ Q) with (P ∧ R → Q).
reflexivity.
apply EqBi.
apply n5_3b.
apply EqBi.
apply H0.
replace (P ∧ Q → P ∧ R) with (P ∧ Q → R).
reflexivity.
apply EqBi.
apply n5_3a.
apply EqBi.
apply H.
apply Equiv4_01.
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 apply Equiv4_01.
 Qed.

Theorem n5_33 : ∀ P Q R : Prop,
 (P ∧ (Q → R)) ↔ (P ∧ ((P ∧ Q) → R)).
 Proof. intros P Q R.
  specialize n5_32 with P (Q→R) ((P∧Q)→R).
  intros n5_32a.
  replace ((P→(Q→R)↔(P∧Q→R))↔(P∧(Q→R)↔P∧(P∧Q→R))) with (((P
→(Q→R)↔(P∧Q→R))→(P∧(Q→R)↔P∧(P∧Q→R)))∧((P∧(Q→R)↔P∧(P∧Q
→R)→(P→(Q→R)↔(P∧Q→R))))) in n5_32a.
  specialize Simp3_26 with ((P→(Q→R)↔(P∧Q→R))→(P∧(Q→R)↔P∧(P∧
Q→R))) ((P∧(Q→R)↔P∧(P∧Q→R)→(P→(Q→R)↔(P∧Q→R)))). (*Not cited
.*)
  intros Simp3_26a.
  MP Simp3_26a n5_32a.
  specialize n4_73 with Q P.
  intros n4_73a.
  specialize n4_84 with Q (Q∧P) R.
  intros n4_84a.
  Syll n4_73a n4_84a Sa.
  replace (Q∧P) with (P∧Q) in Sa.
  MP Simp3_26a Sa.
  apply Simp3_26a.
  apply EqBi.
  apply n4_3. (*Not cited.*)
  apply Equiv4_01.
 Qed.

Theorem n5_35 : ∀ P Q R : Prop,
 ((P → Q) ∧ (P → R)) → (P → (Q ↔ R)).
 Proof. intros P Q R.
```

```
 specialize Comp3_43 with P Q R.
 intros Comp3_43a.
 specialize n5_1 with Q R.
 intros n5_1a.
 specialize Syll2_05 with P (Q∧R) (Q↔R).
 intros Syll2_05a.
 MP Syll2_05a n5_1a.
 Syll Comp3_43a Syll2_05a Sa.
 apply Sa.
 Qed.

Theorem n5_36 : ∀ P Q : Prop,
 (P ∧ (P ↔ Q)) ↔ (Q ∧ (P ↔ Q)).
 Proof. intros P Q.
 specialize n5_32 with (P↔Q) P Q.
 intros n5_32a.
 specialize n2_08 with (P↔Q).
 intros n2_08a.
 replace (P↔Q→P↔Q) with ((P↔Q)∧P↔(P↔Q)∧Q) in n2_08a.
 replace ((P↔Q)∧P) with (P∧(P↔Q)) in n2_08a.
 replace ((P↔Q)∧Q) with (Q∧(P↔Q)) in n2_08a.
 apply n2_08a.
 apply EqBi.
 apply n4_3.
 apply EqBi.
 apply n4_3.
 replace ((P ↔ Q) ∧ P ↔ (P ↔ Q) ∧ Q) with (P ↔ Q → P ↔ Q).
 reflexivity.
 apply EqBi.
 apply n5_32a.
```

```
 Qed. (*The proof sketch cites Ass3_35 and n4_38. Since I couldn't deciph
er how that proof would go, I used a different one invoking other theore
ms.*)

Theorem n5_4 : ∀ P Q : Prop,
 (P → (P → Q)) ↔ (P → Q).
 Proof. intros P Q.
 specialize n2_43 with P Q.
 intros n2_43a.
 specialize n2_02 with (P) (P→Q).
 intros n2_02a.
 Conj n2_43a n2_02a.
 split.
 apply n2_43a.
 apply n2_02a.
 Equiv H.
 apply H.
 apply Equiv4_01.
 Qed.

Theorem n5_41 : ∀ P Q R : Prop,
 ((P → Q) → (P → R)) ↔ (P → Q → R).
 Proof. intros P Q R.
 specialize n2_86 with P Q R.
 intros n2_86a.
 specialize n2_77 with P Q R.
 intros n2_77a.
 Conj n2_86a n2_77a.
 split.
 apply n2_86a.
 apply n2_77a.
 Equiv H.
```

```
  apply H.
  apply Equiv4_01.
  Qed.

Theorem n5_42 : ∀ P Q R : Prop,
  (P → Q → R) ↔ (P → Q → P ∧ R).
  Proof. intros P Q R.
  specialize n5_3 with P Q R.
  intros n5_3a.
  specialize n4_87 with P Q R.
  intros n4_87a.
  replace ((P∧Q)→R) with (P→Q→R) in n5_3a.
  specialize n4_87 with P Q (P∧R).
  intros n4_87b.
  replace ((P∧Q)→(P∧R)) with (P→Q→(P∧R)) in n5_3a.
  apply n5_3a.
  specialize Imp3_31 with P Q (P∧R).
  intros Imp3_31b.
  specialize Exp3_3 with P Q (P∧R).
  intros Exp3_3b.
  Conj Imp3_31b Exp3_3b.
  split.
  apply Imp3_31b.
  apply Exp3_3b.
  Equiv H.
  apply EqBi.
  apply H.
  apply Equiv4_01.
  specialize Imp3_31 with P Q R.
  intros Imp3_31a.
  specialize Exp3_3 with P Q R.
  intros Exp3_3a.
```

```
 Conj Imp3_31a Exp3_3.
 split.
 apply Imp3_31a.
 apply Exp3_3a.
 Equiv H.
 apply EqBi.
 apply H.
 apply Equiv4_01.
 Qed. (*The law n4_87 is really unwieldy to use in Coq. It is actually easie
r to introduce the subformula of the importation-
exportation law required and apply that biconditional. It may be worthw
hile in later parts of PM to prove a derived rule that allows us to manipul
ate a biconditional's subformulas that are biconditionals.*)

Theorem n5_44 : ∀ P Q R : Prop,
 (P→Q) → ((P → R) ↔ (P → (Q ∧ R))).
 Proof. intros P Q R.
 specialize n4_76 with P Q R.
 intros n4_76a.
 replace ((P→Q)∧(P→R)↔(P→Q∧R)) with (((P→Q)∧(P→R)→(P→Q∧R))∧
((P→Q∧R)→(P→Q)∧(P→R))) in n4_76a.
 specialize Simp3_26 with ((P→Q)∧(P→R)→(P→Q∧R)) ((P→Q∧R)→(P→Q
)∧(P→R)).
 intros Simp3_26a. (*Not cited.*)
 MP Simp3_26a n4_76a.
 specialize Exp3_3 with (P→Q) (P→R) (P→Q∧R).
 intros Exp3_3a. (*Not cited.*)
 MP Exp3_3a Simp3_26a.
 specialize Simp3_27 with ((P→Q)∧(P→R)→(P→Q∧R)) ((P→Q∧R)→(P→Q
)∧(P→R)).
 intros Simp3_27a. (*Not cited.*)
 MP Simp3_27a n4_76a.
```

```
 specialize Simp3_26 with (P→R) (P→Q).
 intros Simp3_26b.
 replace ((P→Q)∧(P→R)) with ((P→R)∧(P→Q)) in Simp3_27a.
 Syll Simp3_27a Simp3_26b Sa.
 specialize n2_02 with (P→Q) ((P→Q∧R)→P→R).
 intros n2_02a. (*Not cited.*)
 MP n2_02a Sa.
 clear Sa. clear Simp3_26b. clear Simp3_26a. clear n4_76a. clear Simp3_2
7a.
 Conj Exp3_3a n2_02a.
 split.
 apply Exp3_3a.
 apply n2_02a.
 specialize n4_76 with (P→Q) ((P→R)→(P→(Q∧R))) ((P→(Q∧R))→(P→R)
).
 intros n4_76b.
 replace (((P→Q)→(P→R)→P→Q∧R)∧((P→Q)→(P→Q∧R)→P→R)) with ((
P→Q)→((P→R)→P→Q∧R)∧((P→Q∧R)→P→R)) in H.
 replace (((P→R)→P→Q∧R)∧((P→Q∧R)→P→R)) with ((P→R)↔(P→Q∧R)
) in H.
 apply H.
 apply Equiv4_01.
 replace ((P→Q)→((P→R)→P→Q∧R)∧((P→Q∧R)→P→R)) with (((P→Q)→
(P→R)→P→Q∧R)∧((P→Q)→(P→Q∧R)→P→R)).
 reflexivity.
 apply EqBi.
 apply n4_76b.
 apply EqBi.
 apply n4_3. (*Not cited.*)
 apply Equiv4_01.
 Qed. (*This proof does not use either n5_3 or n5_32. It instead uses four
propositions not cited in the proof sketch, plus a second use of n4_76.*)
```

```
Theorem n5_5 : ∀ P Q : Prop,
  P → ((P → Q) ↔ Q).
  Proof. intros P Q.
  specialize Ass3_35 with P Q.
  intros Ass3_35a.
  specialize Exp3_3 with P (P→Q) Q.
  intros Exp3_3a.
  MP Exp3_3a Ass3_35a.
  specialize n2_02 with P Q.
  intros n2_02a.
  specialize Exp3_3 with P Q (P→Q).
  intros Exp3_3b.
  specialize n3_42 with P Q (P→Q). (*Not mentioned explicitly.*)
  intros n3_42a.
  MP n3_42a n2_02a.
  MP Exp3_3b n3_42a.
  clear n3_42a. clear n2_02a. clear Ass3_35a.
  Conj Exp3_3a Exp3_3b.
  split.
  apply Exp3_3a.
  apply Exp3_3b.
  specialize n3_47 with P P ((P→Q)→Q) (Q→(P→Q)).
  intros n3_47a.
  MP n3_47a H.
  replace (P∧P) with P in n3_47a.
  replace (((P→Q)→Q)∧(Q→(P→Q))) with ((P→Q)↔Q) in n3_47a.
  apply n3_47a.
  apply Equiv4_01.
  apply EqBi.
  apply n4_24. (*with P.*)
  Qed.
```

```
Theorem n5_501 : ∀ P Q : Prop,
  P → (Q ↔ (P ↔ Q)).
  Proof. intros P Q.
  specialize n5_1 with P Q.
  intros n5_1a.
  specialize Exp3_3 with P Q (P↔Q).
  intros Exp3_3a.
  MP Exp3_3a n5_1a.
  specialize Ass3_35 with P Q.
  intros Ass3_35a.
  specialize Simp3_26 with (P∧(P→Q)) (Q→P).
  intros Simp3_26a. (*Not cited.*)
  Syll Simp3_26a Ass3_35a Sa.
  replace ((P∧(P→Q))∧(Q→P)) with (P∧((P→Q)∧(Q→P))) in Sa.
  replace ((P→Q)∧(Q→P)) with (P↔Q) in Sa.
  specialize Exp3_3 with P (P↔Q) Q.
  intros Exp3_3b.
  MP Exp3_3b Sa.
  clear n5_1a. clear Ass3_35a. clear Simp3_26a. clear Sa.
  Conj Exp3_3a Exp3_3b.
  split.
  apply Exp3_3a.
  apply Exp3_3b.
  specialize n4_76 with P (Q→(P↔Q)) ((P↔Q)→Q).
  intros n4_76a. (*Not cited.*)
  replace ((P→Q→P↔Q)∧(P→P↔Q→Q)) with ((P→(Q→P↔Q)∧(P↔Q→Q))
) in H.
  replace ((Q→(P↔Q))∧((P↔Q)→Q)) with (Q↔(P↔Q)) in H.
  apply H.
  apply Equiv4_01.
  replace (P→(Q→P↔Q)∧(P↔Q→Q)) with ((P→Q→P↔Q)∧(P→P↔Q→Q)).
```

```
 reflexivity.
 apply EqBi.
 apply n4_76a.
 apply Equiv4_01.
 replace (P∧(P→Q)∧(Q→P)) with ((P∧(P→Q))∧(Q→P)).
 reflexivity.
 apply EqBi.
 apply n4_32. (*Not cited.*)
 Qed.

Theorem n5_53 : ∀ P Q R S : Prop,
 (((P ∨ Q) ∨ R) → S) ↔ (((P → S) ∧ (Q → S)) ∧ (R → S)).
 Proof. intros P Q R S.
 specialize n4_77 with S (P∨Q) R.
 intros n4_77a.
 specialize n4_77 with S P Q.
 intros n4_77b.
 replace (P ∨ Q → S) with ((P→S)∧(Q→S)) in n4_77a.
 replace ((((P→S)∧(Q→S))∧(R→S))↔(((P∨Q)∨R)→S)) with ((((P∨Q)∨R)
→S)↔(((P→S)∧(Q→S))∧(R→S))) in n4_77a.
 apply n4_77a.
 apply EqBi.
 apply n4_3. (*Not cited explicitly.*)
 apply EqBi.
 apply n4_77b.
 Qed.

Theorem n5_54 : ∀ P Q : Prop,
 ((P ∧ Q) ↔ P) ∨ ((P ∧ Q) ↔ Q).
 Proof. intros P Q.
 specialize n4_73 with P Q.
 intros n4_73a.
```

```
specialize n4_44 with Q P.
intros n4_44a.
specialize Trans2_16 with Q (P↔(P∧Q)).
intros Trans2_16a.
MP n4_73a Trans2_16a.
specialize Trans4_11 with Q (Q∨(P∧Q)).
intros Trans4_11a.
replace (Q∧P) with (P∧Q) in n4_44a.
replace (Q↔Q∨P∧Q) with (~Q↔~(Q∨P∧Q)) in n4_44a.
replace (~Q) with (~(Q∨P∧Q)) in Trans2_16a.
replace (~(Q∨P∧Q)) with (~Q∧~(P∧Q)) in Trans2_16a.
specialize n5_1 with (~Q) (~(P∧Q)).
intros n5_1a.
Syll Trans2_16a n5_1a Sa.
replace (~(P↔P∧Q)→(~Q↔~(P∧Q))) with (~~(P↔P∧Q)∨(~Q↔~(P∧Q
))) in Sa.
replace (~~(P↔P∧Q)) with (P↔P∧Q) in Sa.
specialize Trans4_11 with Q (P∧Q).
intros Trans4_11b.
replace (~Q↔~(P∧Q)) with (Q↔(P∧Q)) in Sa.
replace (Q↔(P∧Q)) with ((P∧Q)↔Q) in Sa.
replace (P↔(P∧Q)) with ((P∧Q)↔P) in Sa.
apply Sa.
apply EqBi.
apply n4_21. (*Not cited.*)
apply EqBi.
apply n4_21.
apply EqBi.
apply Trans4_11b.
apply EqBi.
apply n4_13. (*Not cited.*)
```

```
 replace (~~(P↔P∧Q)∨(~Q↔~(P∧Q))) with (~(P↔P∧Q)→~Q↔~(P∧Q)
).
 reflexivity.
 apply Impl1_01. (*Not cited.*)
 apply EqBi.
 apply n4_56. (*Not cited.*)
 replace (~(Q∨P∧Q)) with (~Q).
 reflexivity.
 apply EqBi.
 apply n4_44a.
 replace (~Q↔~(Q∨P∧Q)) with (Q↔Q∨P∧Q).
 reflexivity.
 apply EqBi.
 apply Trans4_11a.
 apply EqBi.
 apply n4_3. (*Not cited.*)
 Qed.

Theorem n5_55 : ∀ P Q : Prop,
 ((P ∨ Q) ↔ P) ∨ ((P ∨ Q) ↔ Q).
 Proof. intros P Q.
 specialize Add1_3 with (P∧Q) (P).
 intros Add1_3a.
 replace ((P∧Q)∨P) with ((P∨P)∧(Q∨P)) in Add1_3a.
 replace (P∨P) with P in Add1_3a.
 replace (Q∨P) with (P∨Q) in Add1_3a.
 specialize n5_1 with P (P∨Q).
 intros n5_1a.
 Syll Add1_3a n5_1a Sa.
 specialize n4_74 with P Q.
 intros n4_74a.
 specialize Trans2_15 with P (Q↔P∨Q).
```

```
  intros Trans2_15a. (*Not cited.*)
  MP Trans2_15a n4_74a.
  Syll Trans2_15a Sa Sb.
  replace (~(Q↔(P∨Q))→(P↔(P∨Q))) with (~~(Q↔(P∨Q))∨(P↔(P∨Q)))
in Sb.
  replace (~~(Q↔(P∨Q))) with (Q↔(P∨Q)) in Sb.
  replace (Q↔(P∨Q)) with ((P∨Q)↔Q) in Sb.
  replace (P↔(P∨Q)) with ((P∨Q)↔P) in Sb.
  replace ((P∨Q↔Q)∨(P∨Q↔P)) with ((P∨Q↔P)∨(P∨Q↔Q)) in Sb.
  apply Sb.
  apply EqBi.
  apply n4_31. (*Not cited.*)
  apply EqBi.
  apply n4_21. (*Not cited.*)
  apply EqBi.
  apply n4_21.
  apply EqBi.
  apply n4_13. (*Not cited.*)
  replace (~~(Q↔P∨Q)∨(P↔P∨Q)) with (~(Q↔P∨Q)→P↔P∨Q).
  reflexivity.
  apply Impl1_01.
  apply EqBi.
  apply n4_31.
  apply EqBi.
  apply n4_25. (*Not cited.*)
  replace ((P∨P)∧(Q∨P)) with ((P∧Q)∨P).
  reflexivity.
  replace ((P∧Q)∨P) with (P∨(P∧Q)).
  replace (Q∨P) with (P∨Q).
  apply EqBi.
  apply n4_41. (*Not cited.*)
  apply EqBi.
```

```
 apply n4_31.
 apply EqBi.
 apply n4_31.
 Qed.

Theorem n5_6 : ∀ P Q R : Prop,
 ((P ∧ ~Q) → R) ↔ (P → (Q ∨ R)).
 Proof. intros P Q R.
 specialize n4_87 with P (~Q) R.
 intros n4_87a.
 specialize n4_64 with Q R.
 intros n4_64a.
 specialize n4_85 with P Q R.
 intros n4_85a.
 replace (((P ∧ ¬ Q → R) ↔ (P → ¬ Q → R)) ↔ ((¬ Q → P → R) ↔ (¬ Q ∧ P
→ R))) with ((((P ∧ ¬ Q → R) ↔ (P → ¬ Q → R)) → ((¬ Q → P → R) ↔ (¬ Q
∧ P → R)))∧((((¬ Q → P → R) ↔ (¬ Q ∧ P → R)))→(((P ∧ ¬ Q → R) ↔ (P →
¬ Q → R))))) in n4_87a.
 specialize Simp3_27 with (((P ∧ ¬ Q → R) ↔ (P → ¬ Q → R) → (¬ Q → P →
 R) ↔ (¬ Q ∧ P → R))) (((¬ Q → P → R) ↔ (¬ Q ∧ P → R) → (P ∧ ¬ Q → R) ↔
 (P → ¬ Q → R))).
 intros Simp3_27a.
 MP Simp3_27a n4_87a.
 specialize Imp3_31 with (~Q) P R.
 intros Imp3_31a.
 specialize Exp3_3 with (~Q) P R.
 intros Exp3_3a.
 Conj Imp3_31a Exp3_3a.
 split.
 apply Imp3_31a.
 apply Exp3_3a.
 Equiv H.
```

```
MP Simp3_27a H.
 replace (~Q→R) with (Q∨R) in Simp3_27a.
 apply Simp3_27a.
 replace (Q ∨ R) with (¬ Q → R).
 reflexivity.
 apply EqBi.
 apply n4_64a.
 apply Equiv4_01.
 apply Equiv4_01.
 Qed. (*A fair amount of manipulation was needed here to pull the releva
nt biconditional out of the biconditional of biconditionals.*)

Theorem n5_61 : ∀ P Q : Prop,
 ((P ∨ Q) ∧ ~Q) ↔ (P ∧ ~Q).
 Proof. intros P Q.
 specialize n4_74 with Q P.
 intros n4_74a.
 specialize n5_32 with (~Q) P (Q∨P).
 intros n5_32a.
 replace (¬ Q → P ↔ Q ∨ P) with (¬ Q ∧ P ↔ ¬ Q ∧ (Q ∨ P)) in n4_74a.
 replace (~Q∧P) with (P∧~Q) in n4_74a.
 replace (~Q∧(Q∨P)) with ((Q∨P)∧~Q) in n4_74a.
 replace (Q∨P) with (P∨Q) in n4_74a.
 replace (P ∧ ¬ Q ↔ (P ∨ Q) ∧ ¬ Q) with ((P ∨ Q) ∧ ¬ Q ↔ P ∧ ¬ Q) in n4_7
4a.
 apply n4_74a.
 apply EqBi.
 apply n4_3. (*Not cited exlicitly.*)
 apply EqBi.
 apply n4_31. (*Not cited explicitly.*)
 apply EqBi.
 apply n4_3. (*Not cited explicitly.*)
```

```
 apply EqBi.
 apply n4_3. (*Not cited explicitly.*)
 replace (¬ Q ∧ P ↔ ¬ Q ∧ (Q ∨ P)) with (¬ Q → P ↔ Q ∨ P).
 reflexivity.
 apply EqBi.
 apply n5_32a.
 Qed.

Theorem n5_62 : ∀ P Q : Prop,
 ((P ∧ Q) ∨ ~Q) ↔ (P ∨ ~Q).
 Proof. intros P Q.
 specialize n4_7 with Q P.
 intros n4_7a.
 replace (Q→P) with (~Q∨P) in n4_7a.
 replace (Q→(Q∧P)) with (~Q∨(Q∧P)) in n4_7a.
 replace (~Q∨(Q∧P)) with ((Q∧P)∨~Q) in n4_7a.
 replace (~Q∨P) with (P∨~Q) in n4_7a.
 replace (Q∧P) with (P∧Q) in n4_7a.
 replace (P ∨ ¬ Q ↔ P ∧ Q ∨ ¬ Q) with (P ∧ Q ∨ ¬ Q ↔ P ∨ ¬ Q) in n4_7a.
 apply n4_7a.
 apply EqBi.
 apply n4_21. (*Not cited explicitly.*)
 apply EqBi.
 apply n4_3. (*Not cited explicitly.*)
 apply EqBi.
 apply n4_31. (*Not cited explicitly.*)
 apply EqBi.
 apply n4_31. (*Not cited explicitly.*)
 replace (¬ Q ∨ Q ∧ P) with (Q → Q ∧ P).
 reflexivity.
 apply EqBi.
 apply n4_6. (*Not cited explicitly.*)
```

```
  replace (¬ Q ∨ P) with (Q → P).
  reflexivity.
  apply EqBi.
  apply n4_6. (*Not cited explicitly.*)
  Qed.

Theorem n5_63 : ∀ P Q : Prop,
  (P ∨ Q) ↔ (P ∨ (~P ∧ Q)).
  Proof. intros P Q.
  specialize n5_62 with Q (~P).
  intros n5_62a.
  replace (~~P) with P in n5_62a.
  replace (Q ∨ P) with (P ∨ Q) in n5_62a.
  replace ((Q∧~P)∨P) with (P∨(Q∧~P)) in n5_62a.
  replace (P ∨ Q ∧ ~ P ↔ P ∨ Q) with (P ∨ Q ↔ P ∨ Q ∧ ~ P) in n5_62a.
  replace (Q∧~P) with (~P∧Q) in n5_62a.
  apply n5_62a.
  apply EqBi.
  apply n4_3. (*Not cited explicitly.*)
  apply EqBi.
  apply n4_21. (*Not cited explicitly.*)
  apply EqBi.
  apply n4_31. (*Not cited explicitly.*)
  apply EqBi.
  apply n4_31. (*Not cited explicitly.*)
  apply EqBi.
  apply n4_13. (*Not cited explicitly.*)
  Qed.

Theorem n5_7 : ∀ P Q R : Prop,
  ((P ∨ R) ↔ (Q ∨ R)) ↔ (R ∨ (P ↔ Q)).
  Proof. intros P Q R.
```

```
specialize n5_32 with (~R) (~P) (~Q).
intros n5_32a. (*Not cited.*)
replace (~R∧~P) with (~(R∨P)) in n5_32a.
replace (~R∧~Q) with (~(R∨Q)) in n5_32a.
replace ((~(R∨P))↔(~(R∨Q))) with ((R∨P)↔(R∨Q)) in n5_32a.
replace ((~P)↔(~Q)) with (P↔Q) in n5_32a.
replace (~R→(P↔Q)) with (~~R∨(P↔Q)) in n5_32a.
replace (~~R) with R in n5_32a.
replace (R∨P) with (P∨R) in n5_32a.
replace (R∨Q) with (Q∨R) in n5_32a.
replace ((R∨(P↔Q))↔(P∨R↔Q∨R)) with ((P∨R↔Q∨R)↔(R∨(P↔Q))) in
n5_32a.
apply n5_32a. (*Not cited.*)
apply EqBi.
apply n4_21. (*Not cited.*)
apply EqBi.
apply n4_31.
apply EqBi.
apply n4_31.
apply EqBi.
apply n4_13. (*Not cited.*)
replace (~~R∨(P↔Q)) with (~R→P↔Q).
reflexivity.
apply Impl1_01. (*Not cited.*)
apply EqBi.
apply Trans4_11. (*Not cited.*)
apply EqBi.
apply Trans4_11.
replace (~(R∨Q)) with (~R∧~Q).
reflexivity.
apply EqBi.
apply n4_56. (*Not cited.*)
```

```
 replace (~(R∨P)) with (~R∧~P).
 reflexivity.
 apply EqBi.
 apply n4_56.
 Qed. (*The proof sketch was indecipherable, but an easy proof was avail
able through n5_32.*)

Theorem n5_71 : ∀ P Q R : Prop,
 (Q → ~R) → (((P ∨ Q) ∧ R) ↔ (P ∧ R)).
 Proof. intros P Q R.
 specialize n4_4 with R P Q.
 intros n4_4a.
 specialize n4_62 with Q R.
 intros n4_62a.
 specialize n4_51 with Q R.
 intros n4_51a.
 replace (~Q∨~R) with (~(Q∧R)) in n4_62a.
 replace ((Q→~R)↔~(Q∧R)) with (((Q→~R)→~(Q∧R))∧(~(Q∧R)→(Q→
~R))) in n4_62a.
 specialize Simp3_26 with ((Q→~R)→~(Q∧R)) (~(Q∧R)→(Q→~R)).
 intros Simp3_26a.
 MP Simp3_26a n4_62a.
 specialize n4_74 with (Q∧R) (P∧R).
 intros n4_74a.
 Syll Simp3_26a n4_74a Sa.
 replace (R∧P) with (P∧R) in n4_4a.
 replace (R∧Q) with (Q∧R) in n4_4a.
 replace ((P∧R)∨(Q∧R)) with ((Q∧R)∨(P∧R)) in n4_4a.
 replace ((Q∧R)∨(P∧R)) with (R∧(P∨Q)) in Sa.
 replace (R∧(P∨Q)) with ((P∨Q)∧R) in Sa.
 replace ((P∧R)↔((P∨Q)∧R)) with (((P∨Q)∧R)↔(P∧R)) in Sa.
 apply Sa.
```

```
  apply EqBi.
  apply n4_21. (*Not cited.*)
  apply EqBi.
  apply n4_3. (*Not cited.*)
  apply EqBi.
  apply n4_4a. (*Not cited.*)
  apply EqBi.
  apply n4_31. (*Not cited.*)
  apply EqBi.
  apply n4_3. (*Not cited.*)
  apply EqBi.
  apply n4_3. (*Not cited.*)
  apply Equiv4_01.
  apply EqBi.
  apply n4_51a.
  Qed.

Theorem n5_74 : ∀ P Q R : Prop,
  (P → (Q ↔ R)) ↔ ((P → Q) ↔ (P → R)).
  Proof. intros P Q R.
  specialize n5_41 with P Q R.
  intros n5_41a.
  specialize n5_41 with P R Q.
  intros n5_41b.
  Conj n5_41a n5_41b.
  split.
  apply n5_41a.
  apply n5_41b.
  specialize n4_38 with ((P→Q)→(P→R)) ((P→R)→(P→Q)) (P→Q→R) (P→
R→Q).
  intros n4_38a.
  MP n4_38a H.
```

```
  replace (((P→Q)→(P→R))∧((P→R)→(P→Q))) with ((P→Q)↔(P→R)) in n
4_38a.
  specialize n4_76 with P (Q→R) (R→Q).
  intros n4_76a.
  replace ((Q→R)∧(R→Q)) with (Q↔R) in n4_76a.
  replace ((P→Q→R)∧(P→R→Q)) with (P→(Q↔R)) in n4_38a.
  replace (((P→Q)↔(P→R))↔(P→Q↔R)) with ((P→(Q↔R))↔((P→Q)↔(P
→R))) in n4_38a.
  apply n4_38a.
  apply EqBi.
  apply n4_21. (*Not cited.*)
  replace (P→Q↔R) with ((P→Q→R)∧(P→R→Q)).
  reflexivity.
  apply EqBi.
  apply n4_76a.
  apply Equiv4_01.
  apply Equiv4_01.
  Qed.

Theorem n5_75 : ∀ P Q R : Prop,
  ((R → ~Q) ∧ (P ↔ Q ∨ R)) → ((P ∧ ~Q) ↔ R).
  Proof. intros P Q R.
  specialize n5_6 with P Q R.
  intros n5_6a.
  replace ((P∧~Q→R)↔(P→Q∨R)) with (((P∧~Q→R)→(P→Q∨R))∧((P→Q
∨R)→(P∧~Q→R))) in n5_6a.
  specialize Simp3_27 with ((P∧~Q→R)→(P→Q∨R)) ((P→Q∨R)→(P∧~Q→
R)).
  intros Simp3_27a.
  MP Simp3_27a n5_6a.
  specialize Simp3_26 with (P→(Q∨R)) ((Q∨R)→P).
  intros Simp3_26a.
```

```
replace ((P→(Q∨R))∧((Q∨R)→P)) with (P↔(Q∨R)) in Simp3_26a.
Syll Simp3_26a Simp3_27a Sa.
specialize Simp3_27 with (R→~Q) (P↔(Q∨R)).
intros Simp3_27b.
Syll Simp3_27b Sa Sb.
specialize Simp3_27 with (P→(Q∨R)) ((Q∨R)→P).
intros Simp3_27c.
replace ((P→(Q∨R))∧((Q∨R)→P)) with (P↔(Q∨R)) in Simp3_27c.
Syll Simp3_27b Simp3_27c Sc.
specialize n4_77 with P Q R.
intros n4_77a.
replace (Q∨R→P) with ((Q→P)∧(R→P)) in Sc.
specialize Simp3_27 with (Q→P) (R→P).
intros Simp3_27d.
Syll Sa Simp3_27d Sd.
specialize Simp3_26 with (R→~Q) (P↔(Q∨R)).
intros Simp3_26b.
Conj Sd Simp3_26b.
split.
apply Sd.
apply Simp3_26b.
specialize Comp3_43 with ((R→~Q)∧(P↔(Q∨R))) (R→P) (R→~Q).
intros Comp3_43a.
MP Comp3_43a H.
specialize Comp3_43 with R P (~Q).
intros Comp3_43b.
Syll Comp3_43a Comp3_43b Se.
clear n5_6a. clear Simp3_27a. clear Simp3_27b. clear Simp3_27c. clear Si
mp3_27d. clear Simp3_26a. clear Simp3_26b. clear Comp3_43a. clear Co
mp3_43b. clear Sa. clear Sc. clear Sd. clear H. clear n4_77a.
Conj Sb Se.
split.
```

```
  apply Sb.
  apply Se.
  specialize Comp3_43 with ((R→~Q)∧(P↔Q∨R)) (P∧~Q→R) (R→P∧~Q).
  intros Comp3_43c.
  MP Comp3_43c H.
  replace ((P∧~Q→R)∧(R→P∧~Q)) with (P∧~Q↔R) in Comp3_43c.
  apply Comp3_43c.
  apply Equiv4_01.
  apply EqBi.
  apply n4_77a.
  apply Equiv4_01.
  apply Equiv4_01.
  apply Equiv4_01.
  Qed.

End No5.
```